3 (a) $f(x)=\frac{x^3}{3}+\frac{x^4}{4}+x+2$ (b) $f(x)=3x-\frac{x^2}{2}+\frac{3x^4}{2}-1$
(c) $f(x)=x+\frac{1}{x}-2$

4 (a) $V=140t+\frac{13t^2}{2}-\frac{t^3}{3}$ (b) 2040 litres

5 (a) $\frac{d}{dx}\left(xe^x\right)=e^x+xe^x$

(b) $\int xe^x\,dx=\int\left(\frac{d}{dx}\left(xe^x\right)-e^x\right)dx=xe^x-e^x+C$

6 (a) $C-6\cos\frac{x}{2}$ (b) $\frac{x^2}{2}+\frac{1}{2}\tan 2x+C$ (c) $\ln(\sin t)+C$

7 (a) $5\log_e|x|+C$ (b) $2\log_e|x+4|+C$
(c) $2\log_e(x^2+1)+C$ (d) $\log_e(e^x+2)+C$

CHAPTER 17

EXERCISE 17.1

1 $0.2734 < \text{area} < 0.3984$

x	x^2
0	0
0.125	0.015625
0.25	0.0625
0.375	0.140625
0.5	0.25
0.625	0.390625
0.75	0.5625
0.875	0.765625
1	1

2 (a) 10 (b) 9 (c) 8.75
3 (a) 1.5 (b) 1.375 (c) 1.34375
4 (a) 6 (b) 6 (c) 6 5 (a) 8 (b) 5 (c) 4.25

EXERCISE 17.2

1 $\int_0^3(x+2)dx=10.5$ 2 $\int_1^4(2x+1)dx=18$ 3 C

4 $\int_0^t 3x\,dx=\frac{3t^2}{2}$ 5 $\int_0^2 x^3\,dx$, 2.6 6 $\int_0^2(2-x)dx=2$

7 $\int_{-3}^3\sqrt{9-x^2}\,dx=\frac{9\pi}{2}$ 8 $\int_{-1}^5\sqrt{9-(x-2)^2}\,dx=\frac{9\pi}{2}$

EXERCISE 17.3

1 $9,-9;\ 9=-(-9)$ 2 $2\frac{1}{3},6\frac{1}{3},8\frac{2}{3};\ 2\frac{1}{3}+6\frac{1}{3}=8\frac{2}{3}$

3 $8\frac{2}{3},8,16\frac{2}{3};\ 8\frac{2}{3}+8=16\frac{2}{3}$ 4 72, 72; 72 = 72

5 (a) −4 (b) 27.5 (c) $-10\frac{2}{3}$ (d) 0 (e) $6\frac{2}{3}$ (f) $61\frac{2}{3}$ 6 B

7 (a) 0 (b) $18\frac{2}{3}$ (c) 0 (d) $\frac{2}{3}$ (e) $\frac{16}{3}$ (f) $\frac{15}{64}$
(g) 38.4 (h) $\frac{1}{12}$ (i) $-\frac{1}{12}$

8 (a) a^3-b^3 (b) $\frac{8a}{3}+2b+2c$ (c) $\frac{a^3}{3}-a^2$

9 (a) correct (b) incorrect (c) correct (d) correct

10 4.5 11 $\frac{1}{12}$ 12 $14\frac{7}{24}$ 13 $a=2$

14 $a=1$ 15 $c=-\frac{1}{20}$ 16 6.75

EXERCISE 17.4

1 55.5 2 9 3 3 4 $10\frac{2}{3}$

5 (a) correct (b) incorrect (c) correct (d) correct

6 40.5 7 B 8 0.25 9 $k=0.75$

10 (a) $\frac{67}{192}$ (b) $1\frac{1}{3}$ (c) $\frac{5}{12}$

11 $85\frac{1}{3}$ 12 $1\frac{1}{3}$ 13 $3\frac{1}{2}$

14 Area of square = 1 unit^2

(a) $\int_0^1 x\,dx=\left[\frac{x^2}{2}\right]_0^1=\frac{1}{2}$

(b) $\int_0^1 x^2\,dx=\left[\frac{x^3}{3}\right]_0^1=\frac{1}{3}$; $\int_0^1\sqrt{x}\,dx=\left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^1=\frac{2}{3}$;
square is trisected

(c) $\int_0^1 x^3\,dx=\left[\frac{x^4}{4}\right]_0^1=\frac{1}{4}$, $\int_0^1 x\,dx=\left[\frac{x^2}{2}\right]_0^1=\frac{1}{2}$,

$\int_0^1\sqrt[3]{x}\,dx=\left[\frac{3x^{\frac{4}{3}}}{4}\right]_0^1=\frac{3}{4}$; four equal parts

EXERCISE 17.5

1 $\frac{4}{3}$ 2 $21\frac{1}{3}$ 3 A 4 1

5 8 6 $10\frac{2}{3}$ 7 $6\frac{2}{3}$ 8 $\frac{4}{3}$

9 (a) correct (b) correct (c) incorrect (d) correct

10 $\frac{5}{3}$ 11 $\frac{4}{3}$ 12 $10\frac{2}{3}$

13 (a) $y=x$ (b) 4.5 (c) $6\frac{1}{6}$

14 (a) intersection at $\sqrt{x}=\frac{1}{x}$: $x^3=1$, $x=1$; for $x>1$, $\sqrt{x}>\frac{1}{x}$
(b) 1.61

15 (a) $x-20y+480=0$ (b) 16 m (c) $y=\frac{x^2}{200}$ (d) $1983\frac{1}{3}$ m^2

EXERCISE 17.6

1 4.5 2 9 3 C 4 8 5 $10\frac{2}{3}$

6 (a) (2,5): $\frac{dy}{dx}=2x$, $m=4$; $y-5=4(x-2)$, $y=4x-3$

(b) $2\frac{2}{3}$ (c) $1\frac{13}{24}$

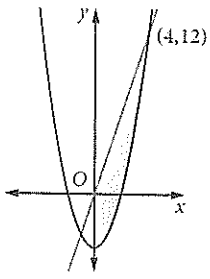
7 $1\frac{11}{24}$

8 (a) $\frac{16\sqrt{2}}{3}$ (b) $\frac{16\left(2-\sqrt{2}\right)}{3}$

9 (a) correct (b) correct (c) correct (d) correct

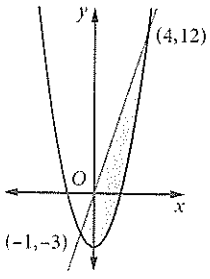
10 4 11 $4\sqrt{6}$

12 (a)



(b) $18\frac{2}{3}$

13 (a)



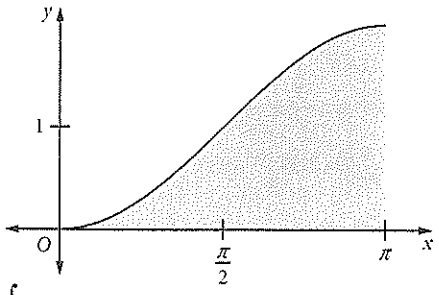
(b) intersection at $(-1,-3)$, $(4,12)$:

$$A = \int_{-1}^{-3} \sqrt{y+4}\,dy - \int_{-1}^{-3} -\sqrt{y+4}\,dy + \int_{-3}^{12}\left(\sqrt{y+4} - \frac{y}{3}\right)dy$$

$$= 2\int_{-1}^{-3} \sqrt{y+4}\,dy + \int_{-3}^{12}\left(\sqrt{y+4} - \frac{y}{3}\right)dy$$

(c) $20\frac{5}{6}$ (d) $\int_{-1}^{4}\left(3x+4-x^2\right)dx = 20\frac{5}{6}$

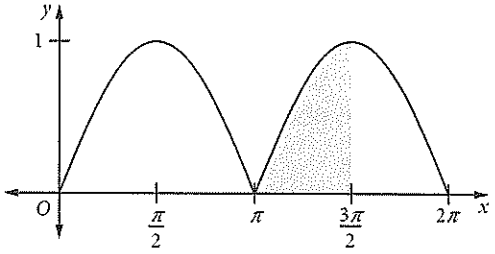
EXERCISE 17.7

1 

$\int$ area $= \pi$

2 (a) 2 (b) $\sqrt{3}$ (c) 1 (d) 1 (e) 2 (f) 0 (g) -0.25
(h) $\frac{\sqrt{2}-1}{2}$ (i) 0 (j) 4 (k) $\frac{-3}{8}$ (l) 0

3 (a) incorrect (b) correct (c) correct (d) correct

4 

area $= 1$

5 2

6 (a) $\log_e 1.5$ (b) $\ln 2$

7 $\frac{-9\sqrt{3}}{8}$ 8 0.5

9 (a) π (b) $\frac{\pi+1}{2}$ (c) $\frac{\pi+2\sqrt{2}}{4}$ (d) $\frac{ab}{\pi}$

10 $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right), \left(\frac{5\pi}{4}, \frac{-\sqrt{2}}{2}\right)$

area $= 4\sqrt{2}$

EXERCISE 17.8

1 (a) $\frac{e^2-1}{e}$ (b) $\frac{e^4-1}{2}$ (c) $\frac{2\left(e^2-1\right)}{e^{\frac{3}{2}}}$ (d) $\frac{2\left(e^{1.5}-1\right)}{3}$
(e) $\frac{e^2-1}{2e}$ (f) $\frac{e^3-1}{3}$ (g) $\frac{\left(e^2-1\right)^2}{2e^2}$ (h) 0 (i) 2.904

2 (a) correct (b) correct (c) correct (d) incorrect

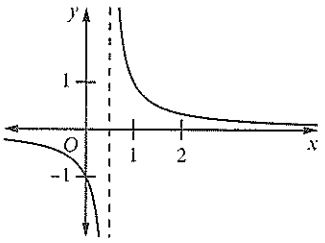
3 $e^2 - e^{-2}$

4 (a) $e^2 - 1$ (b) $y = e^2x - e^2$
(c) $\frac{e^2}{2} - 1$ (d) $e^2 + 1$

5 $e^2 + e^{-2} - 2$ 6 0.511

7 (a) $\ln 2$ (b) $2\ln 2$ (c) $\frac{1}{2}\log_e 5$ (d) $\ln 5$ (e) $\ln 3$
(f) $3\ln 2.5$ (g) $\frac{1}{2}\log_e \frac{7}{3}$ (h) $3\log_e \frac{11}{3}$

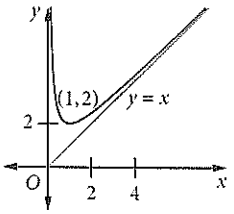
8 (a) $\frac{26+3\ln 3}{3}$ (b) 0 (c) $6 - \ln 2$ (d) $\ln 2 + 0.25$ (e) $\ln 2 + 2.5$
(f) $\int_1^2\left(x^2 - \frac{2}{x} + \frac{1}{x^4}\right)dx = \left[\frac{x^3}{3} - 2\log_e x - \frac{1}{3x^3}\right]_1^2 = \frac{21}{8} - 2\log_e 2$
(g) $e^3 - e + \log_e 3$ (h) $\frac{14}{3} - 2\ln 2$

9 

area $= \int_1^4 \frac{dx}{2x-1} = \frac{\ln 7}{2}$

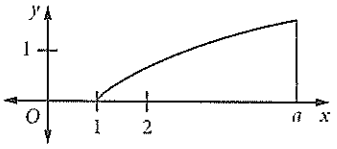
10 $\int_2^4 \frac{x\,dx}{x^2+1} = \frac{1}{2}\left[\ln\left(x^2+1\right)\right]_2^4 = \frac{1}{2}\ln\frac{17}{5}$

11 (a) correct (b) correct (c) incorrect (d) correct

12 

$y' = \frac{x^2-1}{x^2}$, $y'' = \frac{2}{x^3}$,
turning point $(1, 2)$
area $= 1.5 + \ln 2$

13 B 14 $\frac{1+e}{2}$ 15 $\frac{\log_e 2}{2}$

16 

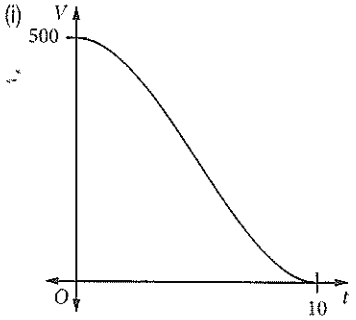
area $= a\ln a - a + 1$

17 (a) $\frac{d}{dx}\left(\log_e(\cos x)\right) = -\tan x$ (b) Area $= \log_e 2$

EXERCISE 17.9

1 (a) $12.5\,\text{cm}^3\,\text{min}^{-1}$ (b) $V = 1000 - 12.5t$, $0 \le t \le 80$

2 (a) (i) 1 item/min (ii) 21 items/min (b) 110

3 (a) (i) $448\,\text{ML}\,\text{h}^{-1}$ (ii) 964 ML
(b) (i) 

(ii) 5 pm
(iii) 2500 ML

4 (a) incorrect (b) correct (c) incorrect (d) correct

5 (a) correct (b) incorrect (c) incorrect (d) correct

6 (a) $x = \frac{t^3}{3} - t^4$ (b) $\ddot{x} = 2t - 12t^2$

7 (a) $x = 2t^3 + 3t^2 - 12t + 7$, $a = 12t + 6$
(b) $t = 1$, $a = 18\,\text{m}\,\text{s}^{-2}$ (c) $v = -12\,\text{m}\,\text{s}^{-1}$, $a = 6\,\text{m}\,\text{s}^{-2}$

8 (a) $\dot{x} = 11 + 10t - t^2$, $x = 11t + 5t^2 - \frac{t^3}{3}$ (b) $t = 11$, $x = 282\frac{1}{3}$

9 (a) $v = 30 - 10t$ (b) $h = 30t - 5t^2$ (c) $t = 3$, $h = 45$ m (d) 6 s

10 (a) $v = 25 - 10t$, $x = 25t - 5t^2$ (b) 31.25 m (c) $t = 1.25$ s

11 (a) $x = t^3 - t^2 - t + 1$, $a = 6t - 2$

(b) $t = 1$ s, $v = 0$ m s^{-1}, $a = 4$ m s^{-2} (c) 4 m

12 $s(t) = 4t^3 - 3t^2 + t + 2$

13 $t = 12$ s, $x = 432$ m

14 (a) 50 km h^{-1}, 20 km h^{-1} (b) 9 min (c) 13.2 km

(d) $x = \frac{112t}{3} + 2$

15 $x = -3$ 16 D

17 (a) $\frac{dx}{dt} = \frac{1}{1+t}$, $x = \int \frac{dt}{1+t} = \log_e(1+t) + C$, $C = 0$, $x = \log_e(1+t)$

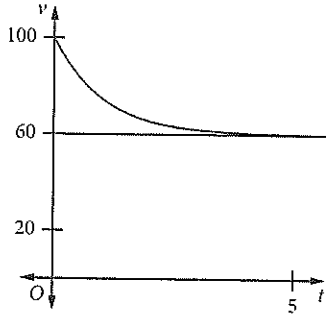
(b) $\lim_{t\to\infty} \log_e(1+t) \to \infty$, moving away in positive direction

(c) -1

18 (a) A: $v = 100$ km h^{-1}; B: $v = 62$ km h^{-1}

(b) (i) $\ddot{x} = -40e^{-t}$ (ii) $\ddot{x} = 60 - v$

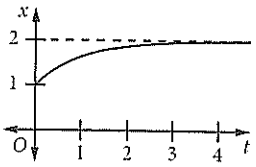
(c)



As t increases, the velocity decreases towards 60 km h^{-1}.

(d) 218 km

19 (a) $x = 2 - e^{-t}$



(b) $\frac{d^2x}{dt^2} = -e^{-t} = x - 2$

20 $a = 2\sin t$, $v = 3 - 2\cos t$, $x = 1 + 3t - 2\sin t$

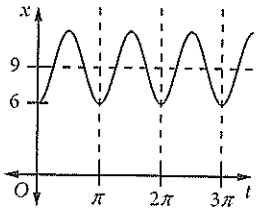
21 (a) $x = 2 + t - 2\cos t$ (b) $t = \log_e 3$

22 (a) $x = 0$ (b) $\dot{x} = 2$

(c) $\ddot{x} = \frac{-4}{(1+t)^2}$; as $(1+t)^2 > 0$ for $t \geq 0$, $\ddot{x} < 0$ for all $t \geq 0$

(d) $t = 1$

23 $v = 6\sin 2t$, $x = 9 - 3\cos 2t$: 6 times



24 (a) $a = \frac{49}{2}e^{-0.5t}$ (b) 0 (c) 49 (d) 155

EXERCISE 17.10

1 $8 + 2\sqrt{13}$ 2 (a) $\frac{31}{40}$ (b) 0.783

3 (a) incorrect (b) correct (c) incorrect (d) correct

4 (a) $3\frac{3}{4}$ (b) $3\frac{15}{16}$

5

x	0	1	2	3	4
2^x	1	2	4	8	16

22.5

6 2.8125 7 (a) $\frac{2}{3}$ (b) 0.631 8 3.414

9 $2\frac{19}{60}$ 10 23.11

11 (a) 63 750 m^2 (b) 159 375 000 m^3

12 (a) $\frac{\pi}{6}\left(\frac{1}{4} + 2 + \frac{1}{4}\right) = \frac{5\pi}{12} \approx 1.31$

(b) 1.31

13 1.57 14 3.1 15 1.54 16 3.65 17 0.188 18 53.44

19 (a) curves do not intersect; $\frac{1}{x^2} > \frac{-1}{x}$ for $1 \leq x \leq 4$

(b) $\frac{1}{2}\left(2 + 2\left(\frac{3}{4} + \frac{4}{9}\right) + \frac{5}{16}\right) \approx 2.35$

EXERCISE 17.11

1 $\frac{1}{3}$ 2 $3\frac{1}{5}$ 3 -2 4 D 5 2

6 $\frac{24}{7}(3 - \sqrt{2})$ 7 3

8 (a) $\frac{e^2 - 1}{2}$ (b) $\frac{e^2 - 1}{2e^2}$ (c) $\frac{e^{10} - e^2}{8}$ (d) $\frac{e^4 - 1}{2e^2}$

9 (a) $\frac{\log_e 3}{2}$ (b) $\frac{\log_e 5}{2}$ (c) $\frac{\log_e 3}{2}$ (d) 0

10 $\frac{4}{\pi}$ 11 $\frac{3}{\pi}$

CHAPTER REVIEW 17

1 (a) -20 (b) $\frac{512}{15}$ (c) 1820 2 $10\frac{2}{3}$

3 (a) 0 (b) $-26\frac{2}{3}$ (c) 48

4 $\frac{2}{3}$ 5 36 6 $42\frac{2}{3}$

7 (a) $\frac{\sqrt{2} - 1}{2}$ (b) $\frac{-\pi^2}{8}$ (c) $\log_e 3$

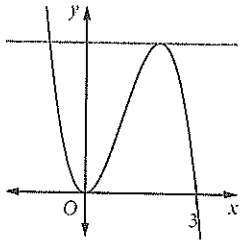
8 $f(x) = \frac{x^3}{3} + x + \frac{35}{24} - \frac{1}{4x}$

9 (a) 6.75 (b) $1\frac{17}{64}$

10 $\frac{19}{6}$ 11 $\frac{\sqrt{2} - \sqrt{3}}{2}$

12 (a) $0 < x < 4$ (b) $3\frac{2}{3}$ (c) 63°26′

13 (a)



(b) $\frac{dy}{dx} = 6x - 3x^2$; at $x = 2$, $\frac{dy}{dx} = 0$; $x = 2$, $y = 4$, $A(2, 4)$

For B, $y = 4$; from the sketch $x = -1$.

Check by substitution: LHS = 3 + 1 = 4 = RHS, so $B(-1, 4)$.

(c) 6.75

14 2.2

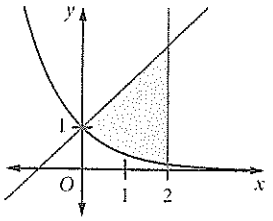
15 (a) 62 m^2

(b) Rotating about the y-axis, the subintervals for the trapezia would not be equal and the curve is not the same above and below the axis. Calculating the volume for each region and taking the average would give an approximate volume:

$$V = \pi\int_0^5 x^2\,dy \approx \pi\left[\frac{2.5}{2}\left(12^2+8^2\right)+\frac{4-2.5}{2}\left(8^2+4^2\right)+\frac{5-4}{2}\left(4^2+0^2\right)\right] \approx 288 \text{ for one side.}$$

16 $-\frac{2}{\pi}$ 17 (a) 0 (b) 24.92 (c) $e^3+\ln 3-e$

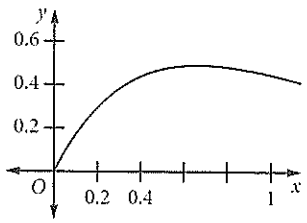
18



area $= \frac{3e^2+1}{e^2}$

19 (a) 0.4905 at $x=\frac{2}{3}$

(b)



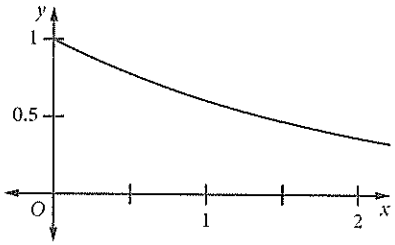
$f(0)=0$,

$f(0.5)=0.472$,

$f(1)=0.446$

(c) 0.348

20 0.7868



21 (a) $x=\frac{\pi}{6}, \frac{7\pi}{6}$ (b) 4

22 (a) $\frac{d}{dx}\left(\log_e(\sin x)\right)=\frac{1}{\sin x}\times\cos x=\cot x$

(b) $\frac{1}{2}\log_e 2-\frac{\pi^2}{32}$

23 (a) $x=1$: $y=\sin\frac{\pi x}{2}=\sin\frac{\pi}{2}=1$; $y=x^2=1^2=1$;
curves meet at $x=1$, $y=1$

(b) $\frac{\pi+6}{3\pi}$

24 $t=2.5\text{ s}$, $x=22\frac{11}{12}\text{ m}$ 25 (a) 14 m s^{-2} (b) 275 m

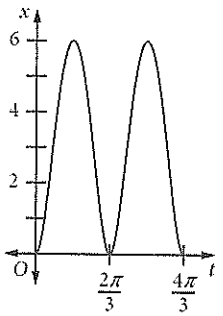
26 (a) $x=2t^3-2t^2+t-10$, $a=12t-4$ (b) 8 m s^{-2} (c) 17 m s^{-1}

27 (a) $t=20\text{ min}$ (b) $V=110t+\frac{17t^2}{2}-\frac{t^3}{3}$ (c) 1968 L

(d) $t=22\text{ min}$ (e) 1015 L

28 (a) $\dot{x}=\int 9\sin 3t\,dt$, $\dot{x}=-3\cos 3t+C$: $t=0$, $v=0$, $C=3$;
$\dot{x}=3-3\cos 3t$

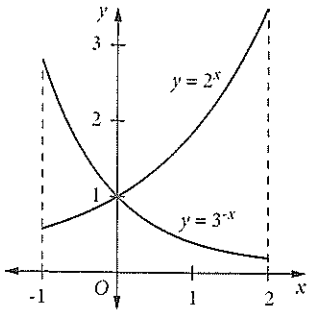
(b)



$t=\frac{2\pi}{3}$.

(c) 2π (d) $x=3t-\sin 3t$

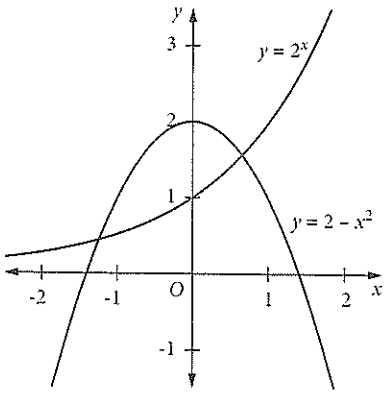
29 (a)



(b) Area $= \int_{-1}^{0}\left(3^{-x}-2^x\right)dx+\int_0^1\left(2^x-3^{-x}\right)dx$

$$=\frac{10}{91\text{n}3}+\frac{5}{21\text{n}2}\approx 4.62 \text{ units}^2$$

30 (a)



Points of intersection (−1.26, 0.42), (0.65, 1.57)

(b) Area $= \int_{-1.26}^{0.65}\left(2-x^2-2^x\right)dx = 1.40 \text{ units}^2$

(c) Area $= \int_{-\sqrt{2}}^{-1.26}\left(2-x^2\right)dx+\int_{-1.26}^{0.65}2^x dx+\int_{0.65}^{\sqrt{2}}\left(2-x^2\right)dx \approx 2.37$

CHAPTER 18

EXERCISE 18.1

1 (a) $A=1000\times 1.003^{12}=\$1036.60$

(b) $A=2000\times 1.005^{10}=\$2102.28$

(c) $A=3400\times 1.0035^{20}=\$3646.08$

2 (a) B (b) D (c) A (d) C

3 $25\,000=P\times 1.0035^{120}$, $P=\frac{25000}{1.0035^{120}}=\$16\,439$