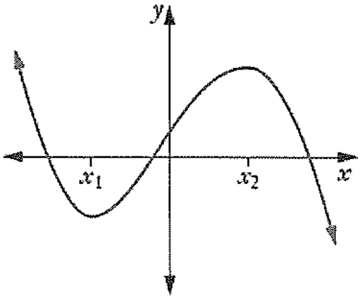
b



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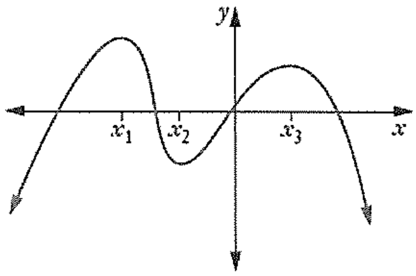


27 $f(x) = 2x^3 - 3x^2 - 31x + 68$

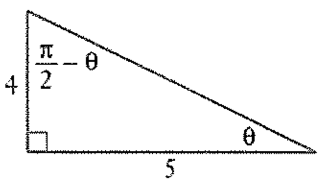
28 $4x - 6\sqrt{3}y + \sqrt{3}\pi - 6 = 0$

29 a $\dfrac{(3x^4 - 5)^7}{84} + C$ **b** $\dfrac{3(x^2 + 1)^{10}}{20} + C$

Challenge exercise 5

1 $2e$

2 a



b 0

c $\theta + \dfrac{\pi}{2} - \theta$

3 $\dfrac{20x^2 - 120x - 1}{(4x^2 + 1)^4}; \dfrac{-24(20x^3 - 140x^2 - 3x + 5)}{(4x^2 + 1)^5}$

4 a $e^{x^2} + C$ **b** $-\dfrac{1}{3}\cos(x^3) + C$

5 $(2x\cos 2x + \sin 2x)e^{x\sin 2x}$

6 $y = \dfrac{x^3}{3} - x^2 - 15x - 1$

7 a $\dfrac{2x}{\sqrt{1 - x^4}}$ **b** $\dfrac{e^x}{1 + e^{2x}}$

c $\dfrac{\cos x - \sin x}{\sin x + \cos x}$

8 $25\frac{5}{6}$

9 $\dfrac{1 + \ln x - x\ln x}{e^x}$

10 a $\theta = \tan^{-1}(4t)$ **b** $3° 32'$

11 a $x = y + e^y$ **b** $P = (1 + e, 1)$

c $x - (1 + e)y = 0$

12 a $\dfrac{\sec^2 x}{\tan x}$ or $\dfrac{1}{\sin x \cos x}$ **b** $-\ln|\cos x| + C$

13 a $-\dfrac{\cos(x^3 - \pi)}{3} + C$ **b** $\dfrac{e^{x^2}}{2} + C$

Chapter 6

Exercise 6.01

1 $x < 2$ **2** $x < \dfrac{1}{4}$

3 $(-\infty, 0)$

4 a $x < 1.5$ **b** $x > 1.5$ **c** $x = 1.5$

5 $f'(x) = -2 < 0$ for all x

6 $y' = 3x^2 > 0$ for all $x \neq 0$

7 $(0, 0)$

8 $x = -3, 2$

9 a $(1, -4)$ **b** $(0, 9)$

c $(1, 1)$ and $(2, 0)$ **d** $(0, 1), (1, 0)$ and $(-1, 0)$

10 $(2, 0)$

11 $-1 < x < 1$

12 $(-\infty, -5) \cup (-3, \infty)$

13 a $x = 2, 5$ **b** $2 < x < 5$ **c** $x < 2, x > 5$

14 $p = -12$

15 $a = 1\frac{1}{2}, b = -6$

16 a $\dfrac{dy}{dx} = 3x^2 - 6x + 27$

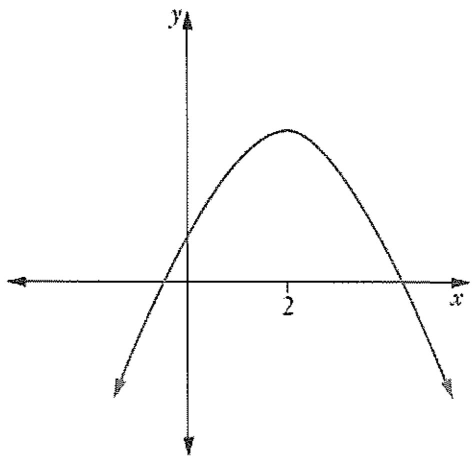
b The quadratic function has $a > 0$

$b^2 - 4ac = -288 < 0$

So $3x^2 - 6x + 27 > 0$ for all x.

The function is monotonic increasing for all x.

17



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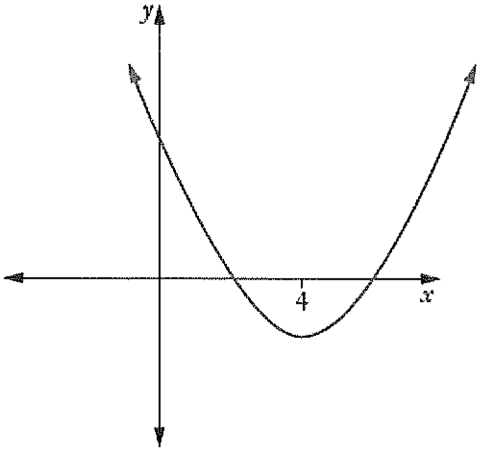

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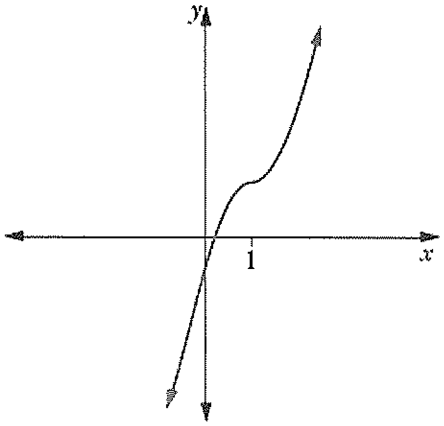

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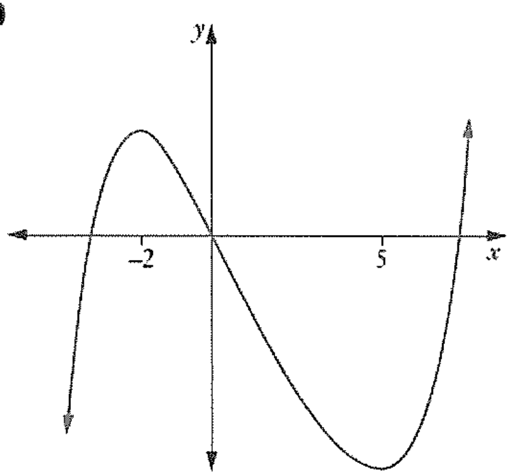

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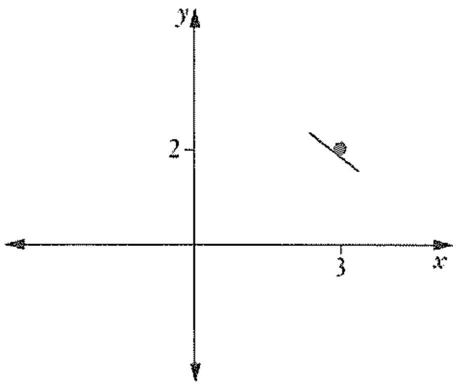

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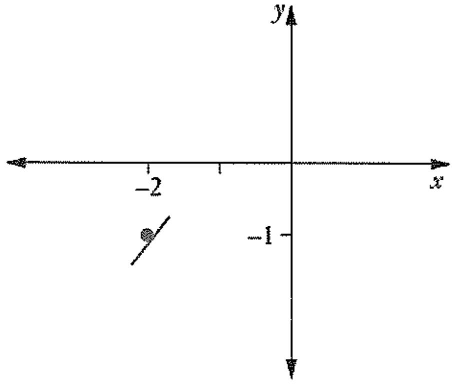


23 $(2, 0)$ and $\left(\frac{2}{3}, 3\frac{13}{81}\right)$

24 $\frac{3x+2}{2\sqrt{x+1}}$; $\left(-\frac{2}{3}, -\frac{2\sqrt{3}}{9}\right)$ **25** $a = -1.75$

26 $y' = \frac{1}{2\sqrt{x}} \neq 0$ **27** $y' = -\frac{3}{x^4} \neq 0$

Exercise 6.02

1 $(0, -1)$; show $y' < 0$ on LHS, $y' > 0$ on RHS

2 $(0, 0)$, minimum

3 $(-2, 11)$; show $f'(x) > 0$ on LHS and $f'(x) < 0$ on RHS

4 $(-1, -2)$, minimum **5** $(4, 0)$ minimum

6 $(0, 5)$ maximum, $(4, -27)$ minimum

7 $(0, 5)$ maximum, $(2, 1)$ minimum

8 $(0, -3)$ maximum, $(1, -4)$ minimum, $(-1, -4)$ minimum

9 $(1, 0)$ minimum, $(-1, 4)$ maximum

10 $m = -3$ **11** $x = -3$ minimum

12 $x = 0$ minimum, $x = -1$ maximum

13 **a** $\frac{dP}{dx} = 2 - \frac{50}{x^2}$

b $(-5, -20)$ maximum, $(5, 20)$ minimum

14 $\left(1, \frac{1}{2}\right)$ minimum

15 $(2.06, 54.94)$ maximum, $(-2.06, -54.94)$ minimum

16 $(4.37, 54.92)$ minimum, $(-4.37, -54.92)$ maximum

17 **a** $\frac{3600 - 2x^2}{\sqrt{3600 - x^2}}$

b $(42.4, 1800)$ maximum, $(-42.4, -1800)$ minimum

Exercise 6.03

1 $x > -\frac{1}{3}$ **2** $x < 3$

3 $y'' = -8 < 0$ **4** $y'' = 2 > 0$

5 $\left(-\infty, 2\frac{1}{3}\right)$ **6** $(1, 9)$

7 $(1, -17)$ and $(-1, -41)$

8 $(0, -2)$: $y'' < 0$ on LHS, $y'' > 0$ on RHS

9 $-2 < x < 1$

10 **a** No

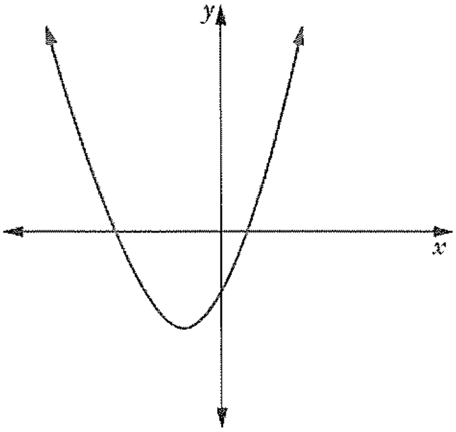
b Yes – point of inflection at (0, 0)

c Yes –point of inflection at (0, 0)

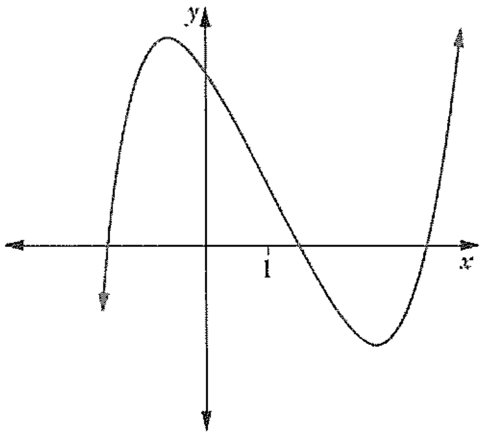
d Yes – point of inflection at (0, 0)

e No

11



12



13 None: (2, 31) is not a point of inflection since concavity does not change.

14 Show that $\frac{12}{x^4} > 0$ for all $x \neq 0$.

15 **a** (0, 7), (1, 0) and (–1, 14) **b** (0, 7)

16 **a** $12x^2 + 24 \neq 0$ and there are no points of inflection.

b The curve is always concave upwards.

17 $a = 2$ **18** $p = 4$ **19** $a = 3, b = -3$

20 **a** (0, –8), (2, 2)

b $\frac{dy}{dx} = 6x^5 - 15x^4 + 21$

At (0, –8):

$\frac{dy}{dx} \neq 0$

At (2, 2):

$\frac{dy}{dx} \neq 0$

Exercise 6.04

1 **a** $\frac{dy}{dx} > 0, \frac{d^2y}{dx^2} > 0$ **b** $\frac{dy}{dx} < 0, \frac{d^2y}{dx^2} < 0$

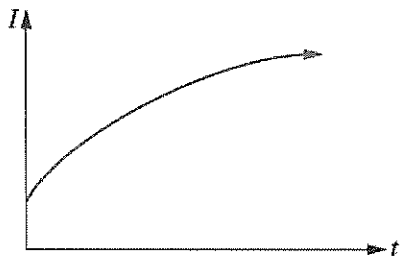
c $\frac{dy}{dx} > 0, \frac{d^2y}{dx^2} < 0$ **d** $\frac{dy}{dx} < 0, \frac{d^2y}{dx^2} > 0$

e $\frac{dy}{dx} > 0, \frac{d^2y}{dx^2} > 0$

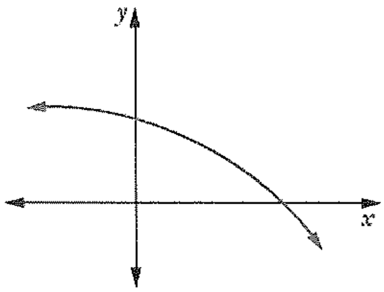
2 **a** $\frac{dP}{dt} > 0, \frac{d^2P}{dt^2} < 0$

b No, the rate is decreasing.

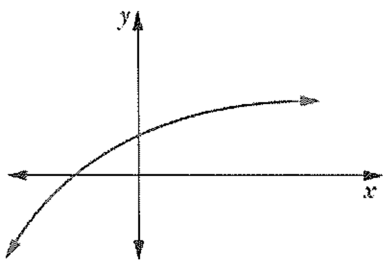
3



4 **a**



b



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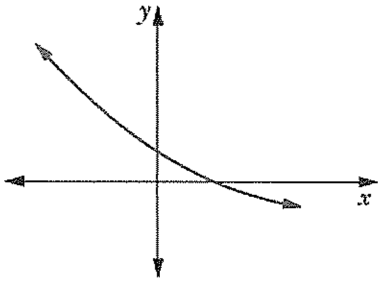
c



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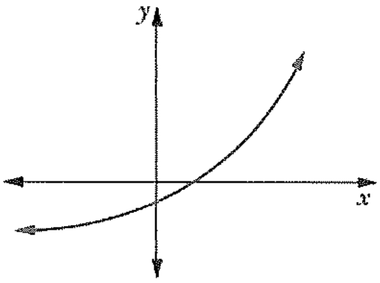

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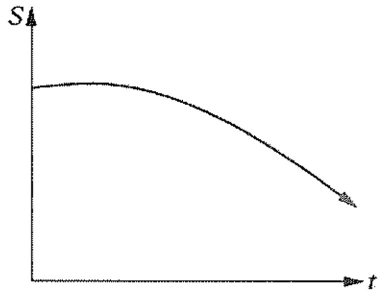

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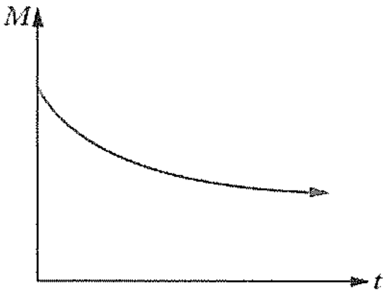
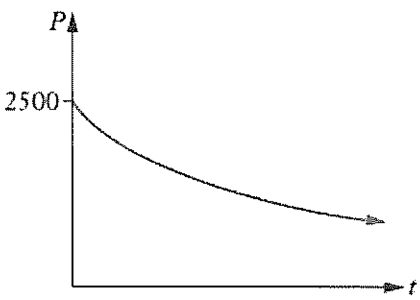


7 $\frac{dM}{dt} < 0, \frac{d^2M}{dt^2} > 0$

8 **a** The number of fish is decreasing.

b The population rate is increasing.

c



9 The level of education is increasing, but the rate of increase is slowing down.

10 The population is decreasing, and the rate of change in population is decreasing.

Exercise 6.05

1 (1, 0), minimum

2 (0, 1), minimum (flat)

3 (2, –5); $y'' = 6 > 0$ so minimum

4 (0.5, 0.25); $y'' = -2 < 0$ so maximum

5 (0, –5); $f''(x) < 0$ on LHS, $f''(x) > 0$ on RHS

6 Yes – point of inflection at (0, 3)

7 (–2, –78) minimum, (–3, –77) maximum

8 (0, 1) maximum, (–1, –4) minimum, (2, –31) minimum

9 (0, 1) maximum, (0.5, 0) minimum, (–0.5, 0) minimum

10 **a** (4, 176) maximum, (5, 175) minimum

b (4.5, 175.5)

11 (3.67, 0.38), maximum

12 (0, –1) minimum, (–2, 15) maximum, (–4, –1) minimum

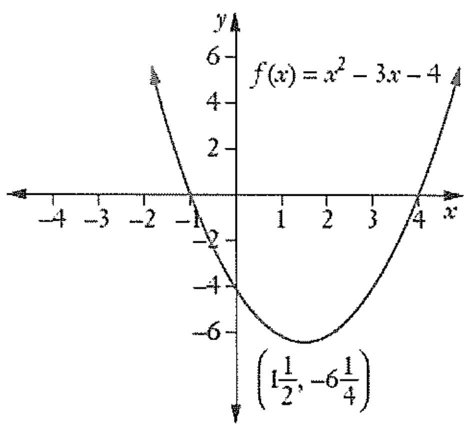
13 **a** $a = 4$ **b** $\left(\frac{1}{2}, 0\right)$, minimum

14 $m = -4$

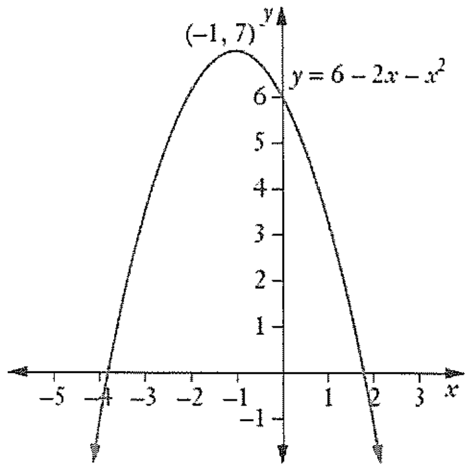
15 $a = 3, b = -9$

Exercise 6.06

1



2



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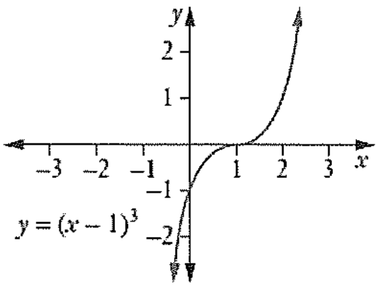
3 (1, 0) point of inflection



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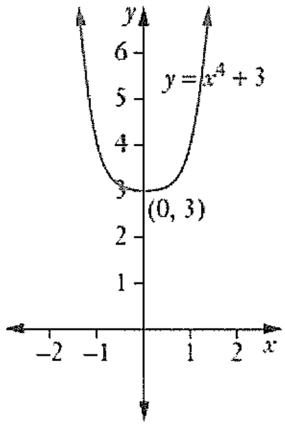
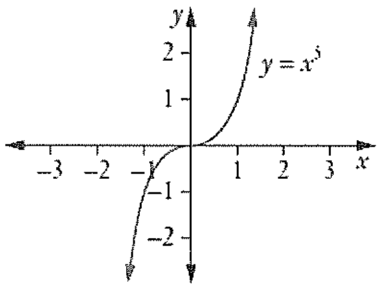
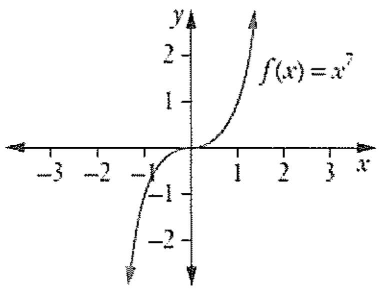


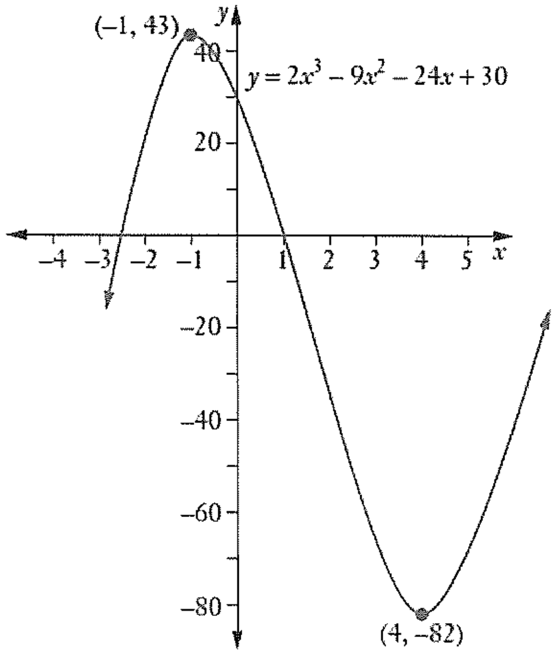
5 (0, 0)



6



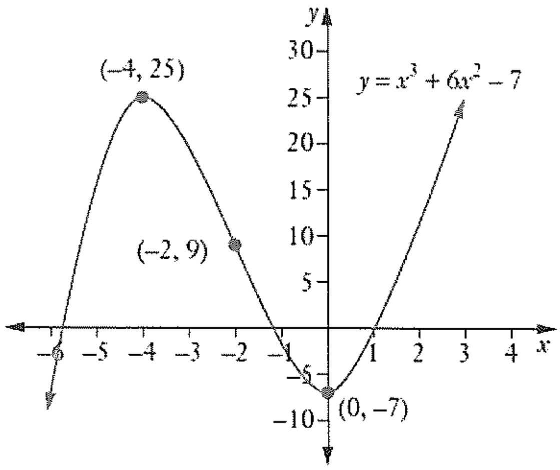
7 (–1, 43) maximum, (4, –82) minimum



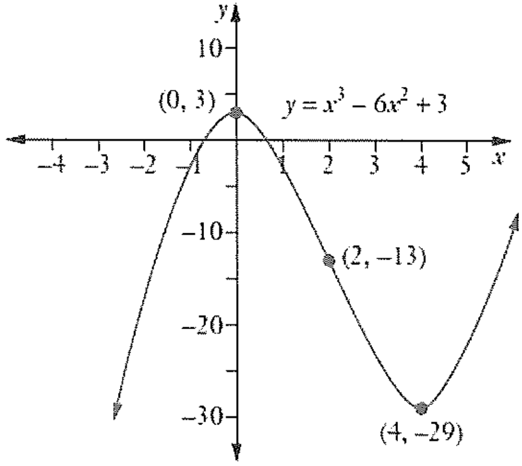
8 **a** (0, –7) minimum, (–4, 25) maximum

b (–2, 9)

c



9 (0, 3) maximum, (2, –13) point of inflection, (4, –29) minimum



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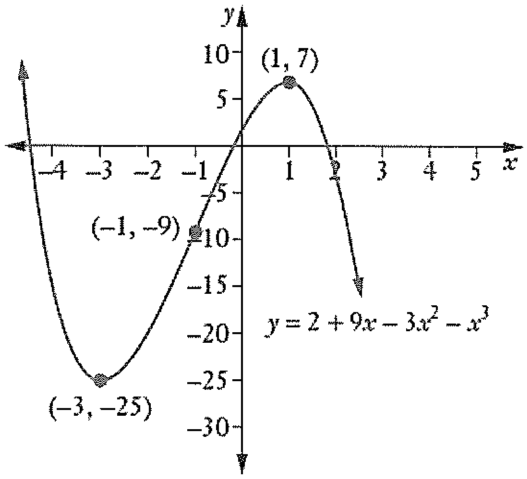
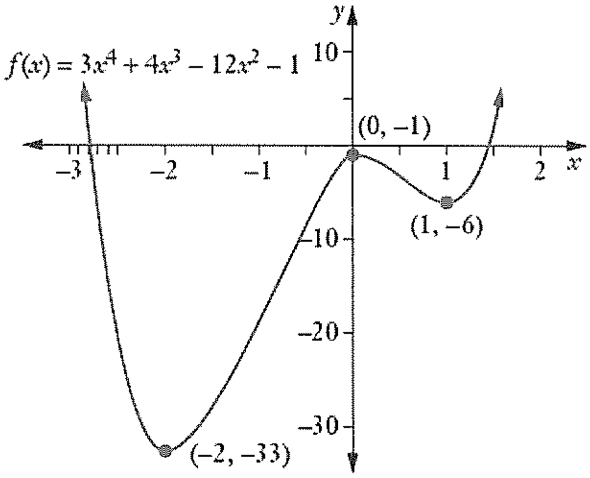
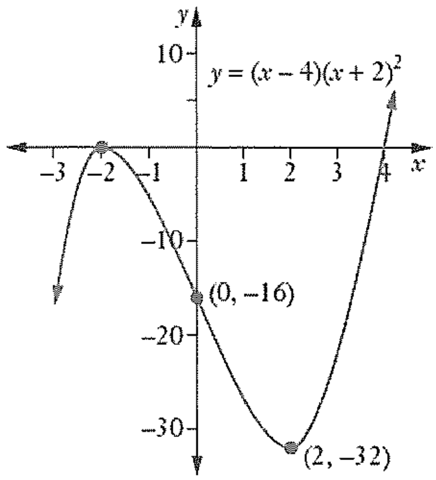
10 $(-3,-25)$ minimum, $(-1,-9)$ point of inflection, $(1, 7)$ maximum



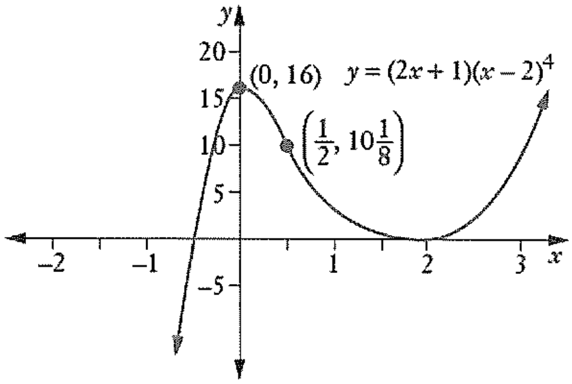
11 $(-2,-33)$ minimum, $(0,-1)$ maximum, $(1,-6)$ minimum



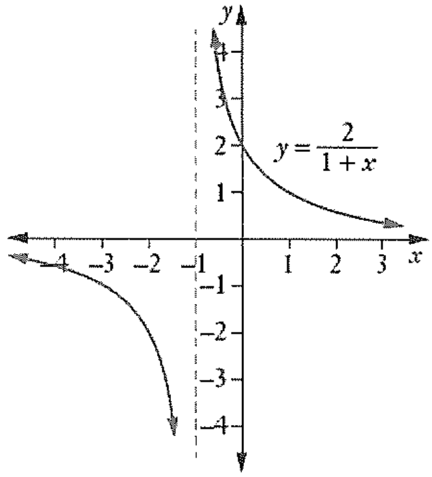
12 $(-2, 0)$ maximum, $(0,-16)$ point of inflection, $(2,-32)$ minimum



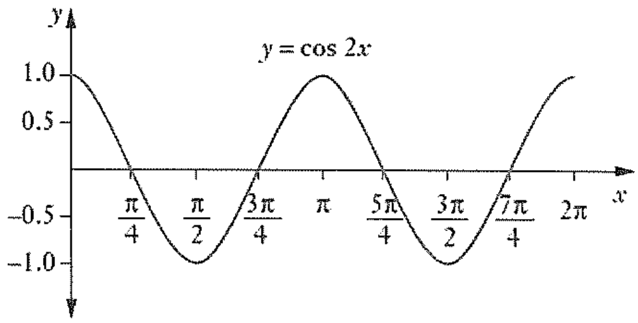
13 $(0, 16)$ maximum, $\left(\frac{1}{2}, 10\frac{1}{8}\right)$ point of inflection, $(2, 0)$ minimum



14 $\frac{dy}{dx} = -\frac{2}{(1+x)^2} \neq 0$ for any x



15 a



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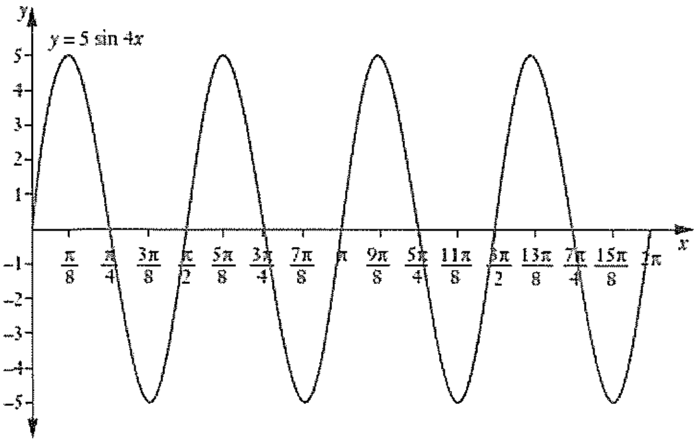
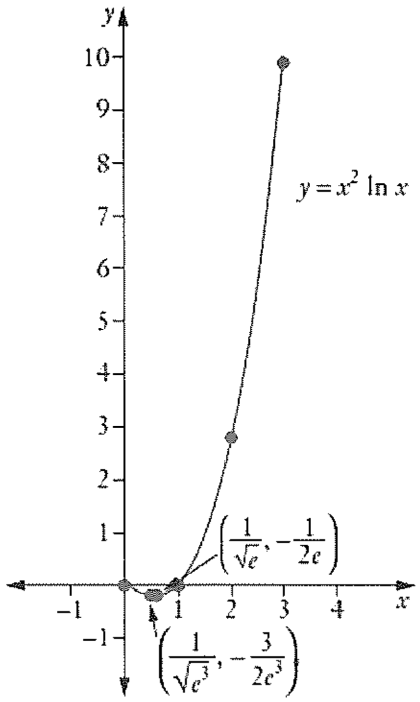


16 a



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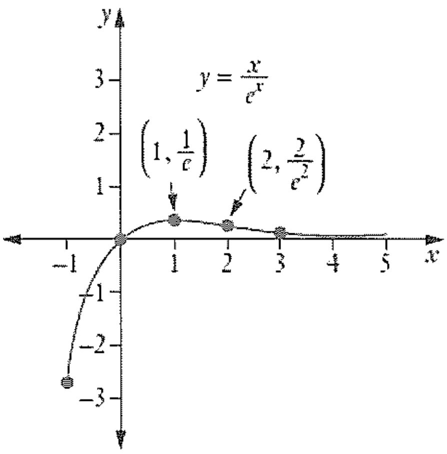

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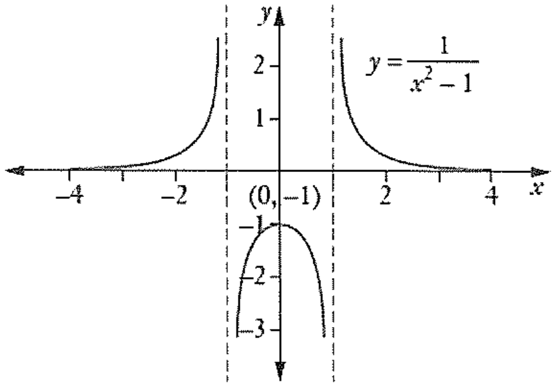
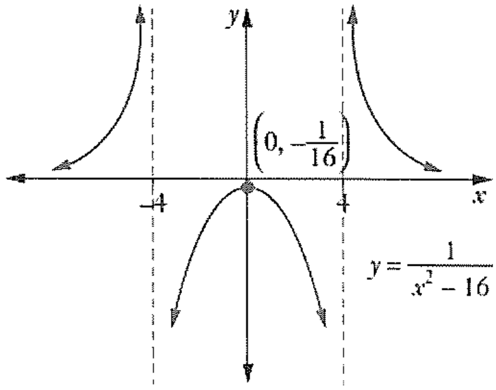


17 a



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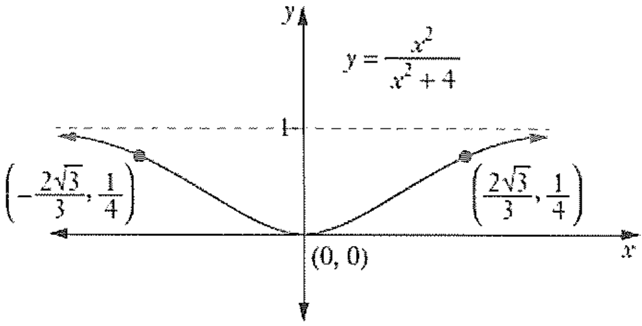

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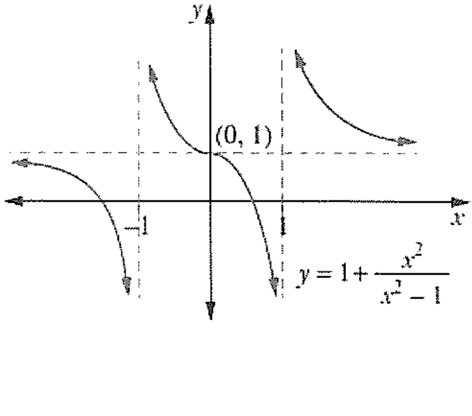

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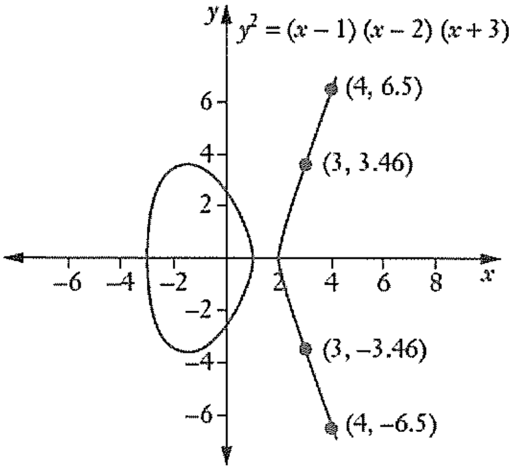
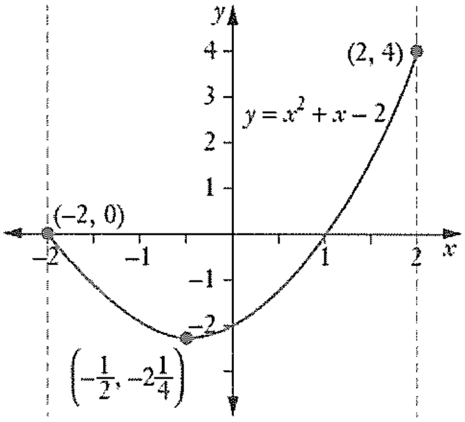
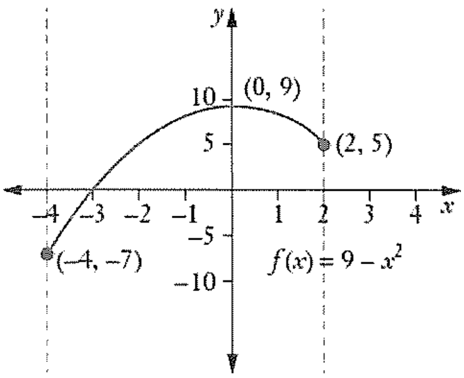


Exercise 6.07

1 Maximum value is 4.

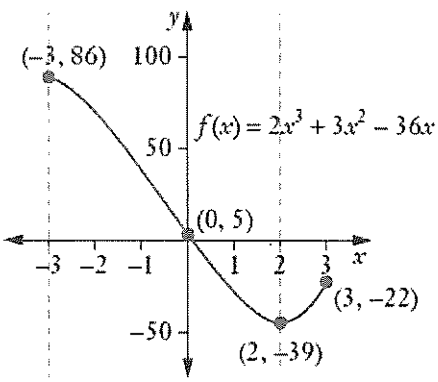


2 Maximum value is 9, minimum value is −7.



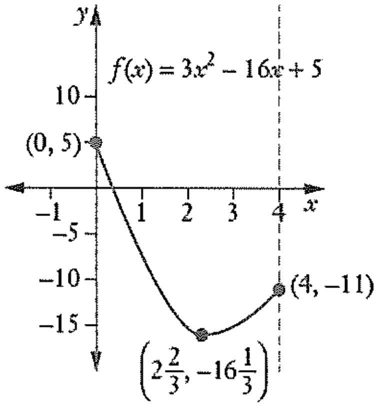
3 Maximum value is 25.

4 Maximum value is 86, minimum value is −39.



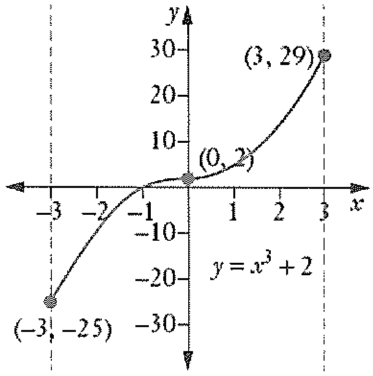
5 Maximum value is −2.

6 Maximum value is 5, minimum value is $-16\frac{1}{3}$.

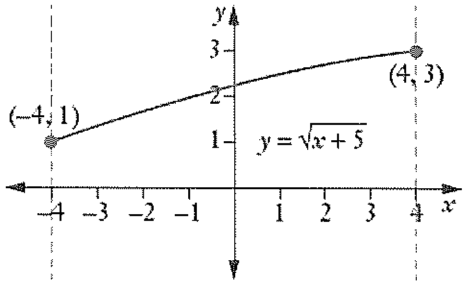


7 Global maximum 29, local maximum −3, global minimum −35, local minimum −35, −8

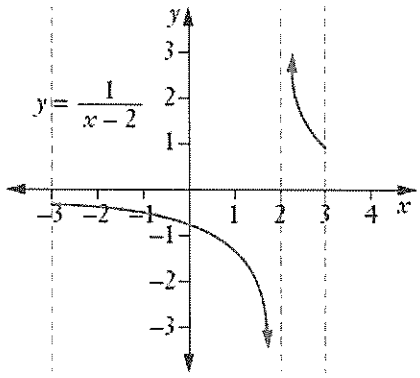
8 Minimum −25, maximum 29



9 Maximum 3, minimum 1



10 Maximum ∞, minimum −∞



Investigation

The disc has radius $\frac{30}{7}$ cm. (This result uses Stewart's theorem – research this.)

Exercise 6.08

See worked solutions for full proofs.

1 $\frac{50}{x} = y$

$P = 2x + 2y$

2 $y = 60 - x$

$A = xy$

3 $\frac{20}{x} = y$

$S = x + y$

4 $\frac{400}{\pi r^2} = h$

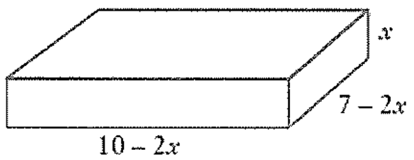
$S = 2\pi r^2 + 2\pi rh$

5 **a** $x + y = 30$

b $A = \left(\frac{1}{4}x\right)^2 + \left(\frac{1}{4}y\right)^2$

6 **a** $x^2 + y^2 = 280^2 = 78\ 400$ **b** $A = xy$

7

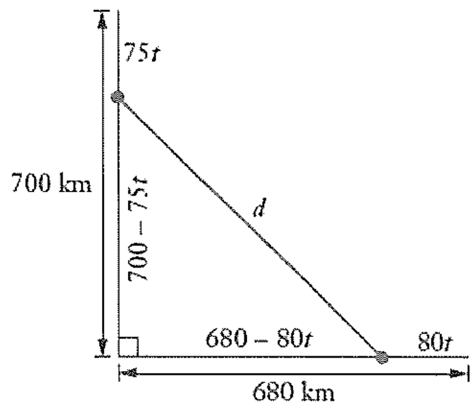


$V = x(10 - 2x)(7 - 2x)$

8 Profit per person = Cost – Expenses

$= (900 - 100x) - (200 + 400x)$

9



$d^2 = (700 - 75t)^2 + (680 - 80t)^2$

10 Distance AB: $d = \sqrt{x^2 + 0.5^2}$

$t = \frac{\sqrt{x^2 + 0.25}}{5}$

Distance BC: $d = 7 - x$

$t = \frac{7 - x}{4}$

Exercise 6.09

See worked solutions for full proofs.

1 2 s, 16 m **2** 7.5 km

3 **a** $y = 30 - x$

b Maximum area is 225 m^2

4 **a** $\frac{4000}{x} = y$

$P = 2x + 2y$

b 63.2 m by 63.2 m **c** \$12 322.88

5 4 m by 4 m **6** 14 and 14

7 –2.5 and 2.5

8 $x = 1.25$ m, $y = 1.25$ m

9 **a** $V = x(30 - 2x)(80 - 2x)$

b $x = 6\frac{2}{3}$cm **c** 7407.4 cm^3

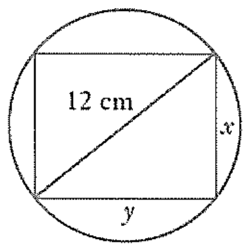
10 **a** $\frac{54}{r^2} = h$

$S = 2\pi r(r + h)$

b Radius is 3 m.

11 **a** $S = 2\pi r^2 + \frac{17\ 200}{r}$ **b** 2324 m^2

12 **a**



$x^2 + y^2 = 12^2$

$A = xy = x\sqrt{144 - x^2}$

b 72 cm^2

13 **a** $\frac{400}{x} = y$

$A = (x - 10)(y - 10)$

b 100 cm^2

14 1.12 m^2

15 **a** 7.5 m by 7.5 m **b** 2.4 m

16 160.17 cm^2

17 1.68 m, 1.32 m

18 **a** $d^2 = (200 - 80t)^2 + (120 - 60t)^2$

b Minimum distance 24 km

19 **a** $d = (x^2 - 2x + 5) - (4x - x^2)$

$= 2x^2 - 6x + 5$

b 0.5

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20 **a** $s = \dfrac{d}{t}$

So $t = \dfrac{d}{s}$

$= \dfrac{1500}{s}$

Cost of trip taking t hours:

$C = (s^2 + 9000)t$

$= (s^2 + 9000)\dfrac{1500}{s}$

$= 1500\left(s + \dfrac{9000}{s}\right)$

b 95 km h^{-1} **c** \$2846

Test yourself 6

1 A **2** C **3** D **4** C

5 (–3, –11) maximum, (–1, –15) minimum

6 $x > 1\frac{1}{6}$

7 50 m

8 $x > -1$

9 $(\frac{1}{2}, -1)$

10 **a** $\dfrac{375}{\pi r^2} = h$

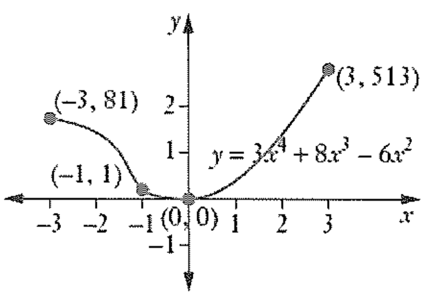
$S = 2\pi r^2 + 2\pi rh$

b 3.9 cm

11 **a** (0, 0) and (–1, 1)

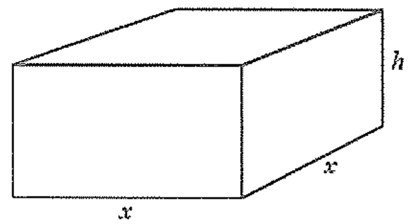
b (0, 0) minimum, (–1, 1) horizontal point of inflection

c



d Maximum value 513, minimum value 0

12 **a**



$\dfrac{125 - x^2}{2x} = h$

$V = x^2 h$

b 6.45 cm by 6.45 cm by 6.45 cm

13 150 products

14 **a** $y = \sqrt{25 - x^2}$

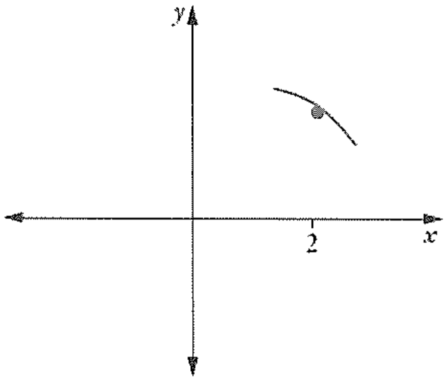
$A = \dfrac{1}{2}xy$

b 6.25 m^2

15 (0, 1) and (3, –74)

16 179

17



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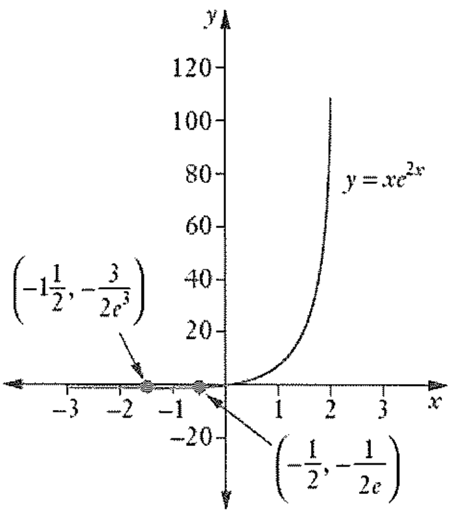

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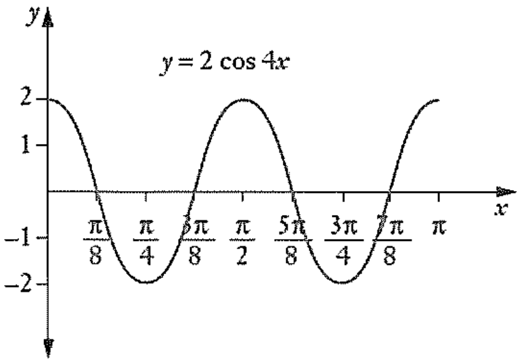
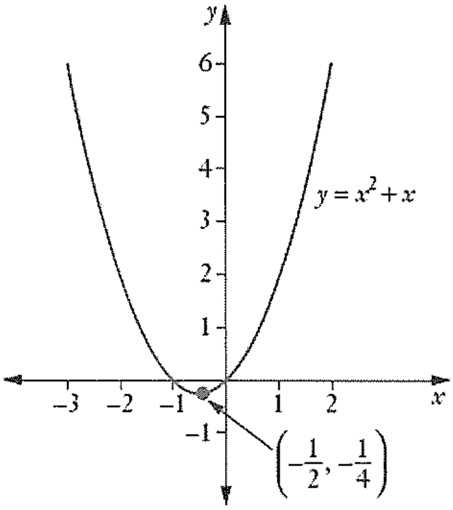
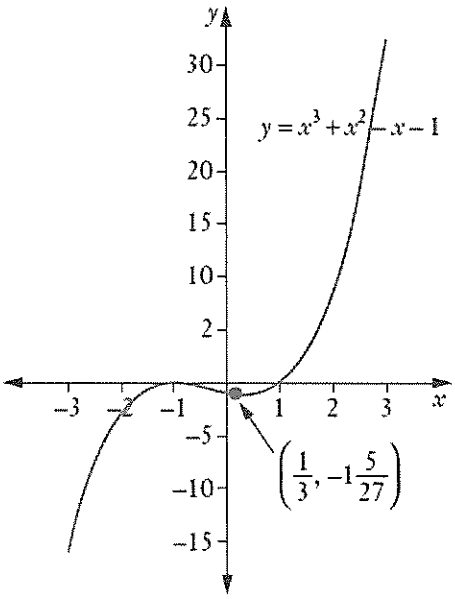


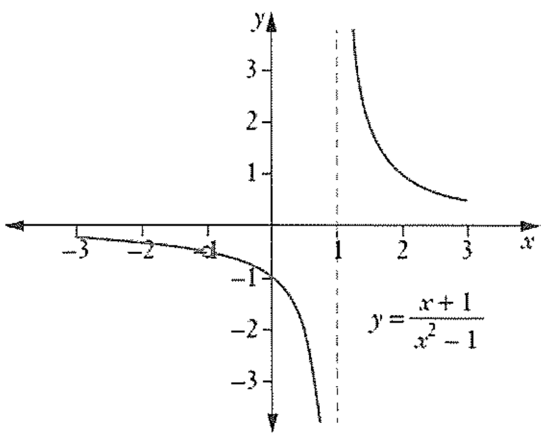
20 a



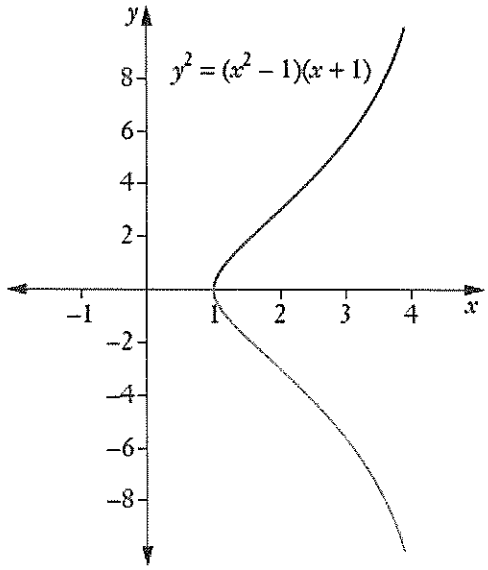
b



c

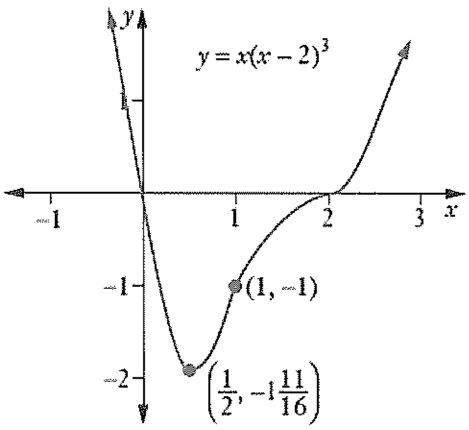


d



Challenge exercise 6

1



2 $x < -\frac{1}{2}, x > 4$

3 16 m^2

4 $27; -20.25$

5 $f'(0.6) = f'(0.6) = 0$ and concavity changes

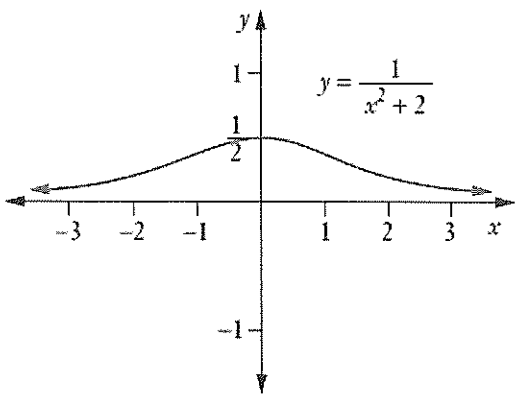
6 Proof. See Worked Solutions. $r = s = 12.5$

7 $y = x^2 + 2x + 3$

8 a $\left(0, \frac{1}{2}\right)$, maximum

b Domain: $(-\infty, \infty)$, range: $\left(0, \frac{1}{2}\right]$

c $0^+, 0^+$



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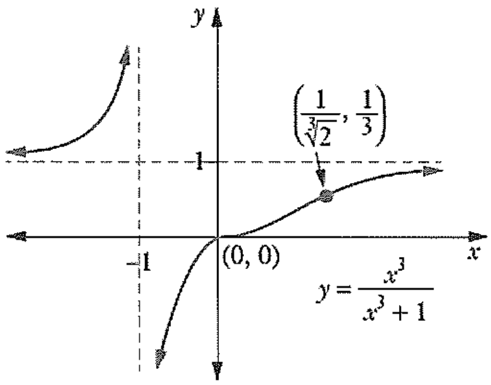
9 a $y' = 0$ at $(0, 0)$
b $y'' > 0$ on LHS and RHS
c $y'' < 0$ on LHS, $y'' > 0$ on RHS

10

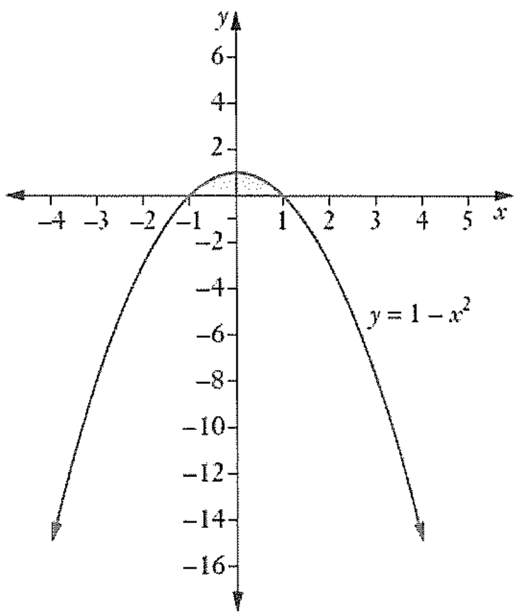


11 Minimum –1, maximum $-\frac{1}{5}$
12 87 km h^{-1}

Chapter 7

Exercise 7.01

1 a 4.125 $units^2$ b 6.625 $units^2$
2 a 1.17 $units^2$ b 1.67 $units^2$ c 1.27 $units^2$
d 1.52 $units^2$
3 a 6.5 $units^2$ b 5 $units^2$ c 132 $units^2$
d $\frac{\pi(1+\sqrt{2})}{8\sqrt{2}} = \frac{\pi}{16}(2+\sqrt{2})$ $units^2$ e 6.5 $units^2$
4 a 28 $units^2$ b 156 $units^2$ c 140 $units^2$
5 a $3\frac{3}{7}$ $units^2$ b 5.5 $units^2$ c $\frac{\pi}{4}$ $units^2$
d $\frac{3}{2}(e + e^4)$ e 23 $units^2$
6 a



b 1 $unit^2$
7 a 4.41 $units^2$ b 4.91 $units^2$
c 4.5 $units^2$ d 4.5 $units^2$
8 $\frac{25\pi}{2}$ $units^2$
9 a $\frac{9\pi}{2}$ $units^2$
b i 2.5 $units^2$ ii 8.1 $units^2$
10 a 22.4 $units^2$ b 3.3 $units^2$ c 1 $unit^2$
11 a i 6 $units^2$ ii 14 $units^2$
b i $\frac{\pi\sqrt{2}}{4}$ $units^2$ ii $\frac{\pi(\sqrt{2}+2)}{4}$ $units^2$
12 a 60.625 $units^2$ b 73.125 $units^2$

Exercise 7.02

1 a 2.5 b 10
c 2.4 d 0.225
2 a 28 b 22
3 a 0.39 b 0.41
4 a 1.08 b 0.75 c 0.65
d 0.94 e 0.92
5 a 75.1 b 16.5 c 650.2
6 a 28.9 m^2 b 39.25 m^2 c 7.45 km^2
d 492.25 m^2

Exercise 7.03

1 a 8 b 10 c 217
d –1 e 10 f 54
g $3\frac{1}{3}$ h 16 i 50
2 a $\frac{2}{3}$ b $21\frac{1}{4}$ c 0
d $4\frac{2}{3}$ e $1\frac{1}{4}$ f $4\frac{1}{3}$
g 0 h $2\frac{1}{3}$ i 0
j $6\frac{2}{9}$ k 100 l 54
m $15\frac{5}{6}$ n $22\frac{2}{3}$ o 0.0126