(b) $\left(\frac{\pi}{4}, 2.028\right), \left(\frac{5\pi}{4}, 0.493\right)$

$e^{\sin x} = e^{\cos x}$ $\sin x = \cos x$, $\tan x = 1$,

$x = \frac{\pi}{4}, \frac{5\pi}{4}$, $f\left(\frac{\pi}{4}\right) = e^{\frac{1}{\sqrt{2}}} = 2.028$, $f\left(\frac{5\pi}{4}\right) = e^{\frac{-1}{\sqrt{2}}} = 0.493$

(c) $f'(x) = \cos x\, e^{\sin x}$, $g'(x) = -\sin x\, e^{\cos x}$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} e^{\frac{1}{\sqrt{2}}}$, $g'\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} e^{\frac{1}{\sqrt{2}}}$

$f'\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} e^{\frac{-1}{\sqrt{2}}}$, $g'\left(\frac{5\pi}{4}\right) = \frac{1}{\sqrt{2}} e^{\frac{-1}{\sqrt{2}}}$

(d) No

$g'\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} e^{\frac{1}{\sqrt{2}}} = -f'\left(\frac{\pi}{4}\right)$

$g'\left(\frac{5\pi}{4}\right) = \frac{1}{\sqrt{2}} e^{\frac{-1}{\sqrt{2}}} = -f'\left(\frac{5\pi}{4}\right)$

5 0

CHAPTER REVIEW 13

1 (a) $\cos x + 2\sec^2 2x$ (b) $-12\sin 4x - 10\cos 2x$

(c) $\cos x - x\sin x$ (d) $\frac{-1}{\sin^2 x} = -\text{cosec}^2\, x$

(e) $-2e^{-x}(\cos 3x + 3\sin 3x)$ (f) $\frac{2\cos 2x}{\sin 2x} = 2\cot 2x$

2 (a) $(x^2 + 4x + 2)e^x$ (b) $\frac{2e^{-x}(1 - x\ln x)}{x}$ (c) $\frac{e^x}{1 + e^x}$

(d) $\frac{2x+2}{x^2 + 2x}$ (e) $e^{-3x}(3 - 7x - 3x^2)$ (f) $\frac{1}{2\sqrt{x}} e^{\sqrt{x}} + \frac{1}{2x}$

3 (a) 9, 6, 0, −3, 0, 6, 9 (b) $t = 4$, $v = \frac{-\pi\sqrt{3}}{2}$, $a = \frac{\pi^2}{12}$

4 (a) $2x\sin x + x^2\cos x$ (b) $\frac{2x\cos x + x^2\sin x}{\cos^2 x}$

(c) $\cos x\cos x + \sin x(-\sin x) = \cos^2 x - \sin^2 x$

(d) $\frac{\sec^2 x}{2\sqrt{\tan x}}$

(e) $2x \times (-\sin(x^2)) = -2x\sin(x^2)$

(f) $\frac{(2x+1)\cos x - 2\sin x}{(2x+1)^2}$

5 (a) $\frac{2x+2}{x^2 + 2x + 1} = \frac{2(x+1)}{(x+1)^2} = \frac{2}{x+1}$

(b) $e^x \log_e x + \frac{e^x}{x}$ (c) $\frac{\frac{1}{x} \times e^x - \log_e x \times e^x}{e^{2x}} = \frac{1 - x\log_e x}{x e^x}$

(d) $\frac{1}{\tan x} \times \sec^2 x = \frac{1}{\sin x\cos x}$

(e) $\frac{\left(2x + \frac{1}{x}\right) \times x - \left(x^2 + \log_e x\right) \times 1}{x^2} = \frac{x^2 + 1 - \log_e x}{x^2}$

(f) $4x^3 - 2x\sin(x^2) + \cot x$

6 (a) $f(x) = \log_e x + \log_e(\tan x)$

$f'(x) = \frac{1}{x} + \frac{\sec^2 x}{\tan x} = \frac{1}{x} + \frac{1}{\sin x\cos x}$

(b) $y = \log_e(x^3 - 6) - \log_e(e^{-x} - 1)$

$\frac{dy}{dx} = \frac{3x^2}{x^3 - 6} + \frac{e^{-x}}{e^{-x} - 1}$

(c) $f(x) = \frac{1}{2}\log_e x + \log_e(\cos x) - \log_e(1 - \sin^2 x)$

$f'(x) = \frac{1}{2x} - \frac{\sin x}{\cos x} - \frac{-2\sin x\cos x}{1 - \sin^2 x} = \frac{1}{2x} + \tan x$

OR

$f(x) = \log_e \frac{\sqrt{x}\cos x}{1 - \sin^2 x} = \log_e \frac{\sqrt{x}\cos x}{\cos^2 x} = \log_e x - \log_e(\cos x)$

$f'(x) = \frac{1}{2x} - \frac{-\sin x}{\cos x} = \frac{1}{2x} + \tan x$

7 (a) $x^2\, 10^x(3 + x\ln 10)$ (b) $\cos x + \frac{1}{x\ln a}$

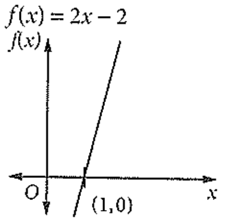
(c) $2^x \ln 2 + 3^x \ln 3 + 4^x \ln 4$ (d) $\frac{a^x\left(x(\ln a)^2 \log_a x - 1\right)}{x\ln a\left(\log_a x\right)^2}$

CHAPTER 14

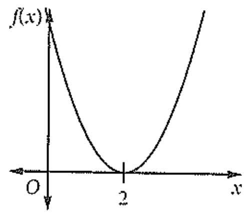
EXERCISE 14.1

1 A, C

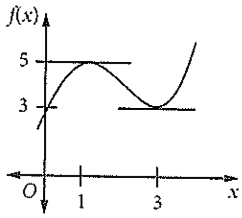
2 $f(x) = 2x - 2$



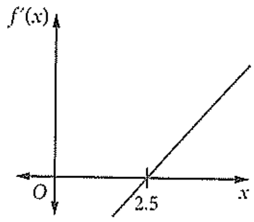
3



4



5

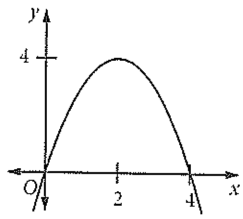


$f'(x) = 2x - 5$

(a) $x < 2.5$ (b) $x = 2.5$

(c) $x > 2.5$

6



$x = 2$; positive, negative

7 (a) $x < -1.5$ (b) $x > -1.5$ (c) $x = -1.5$

8 (a) $f'(x) = 3x^2 - 12x + 9$ (b) $x < 1, x > 3$

(c) $1 < x < 3$ (d) $x = 1$

9 (a) real x (b) none (c) never

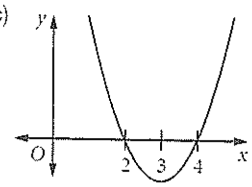
10 (a) $x = -\frac{1}{3}, 1$ (b) $x < -\frac{1}{3}, x > 1$ (c) $-\frac{1}{3} < x < 1$

EXERCISE 14.2

1 (a) $f'(x) = 2x - 6$

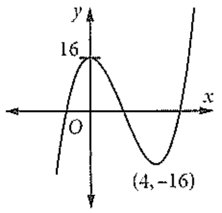
(b) $(3, -1)$ minimum turning point

(c)



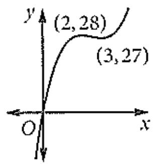
2 C

3 (a) $f'(x) = 3x^2 - 12x$
(b) $(0, 16)$ local maximum; $(4, -16)$ local minimum
(c)

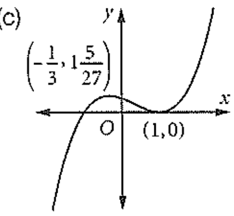


4 (a) correct (b) incorrect (c) correct (d) correct

5 (a) $f'(x) = 6x^2 - 30x + 36$
(b) $(2, 28)$ local maximum;
$(3, 27)$ local minimum
(c)



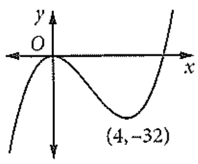
6 (a) $f'(x) = 3x^2 - 2x - 1$
(b) $-\frac{1}{3}, 1\frac{5}{27}$ local maximum;
$(1, 0)$ local minimum
(c)



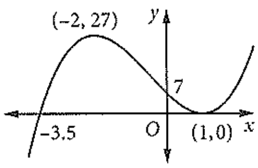
7 $3\frac{1}{8}$

8 2

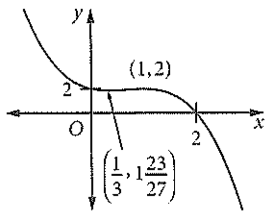
9



10 $(-2, 27)$ maximum;
$(1, 0)$ minimum

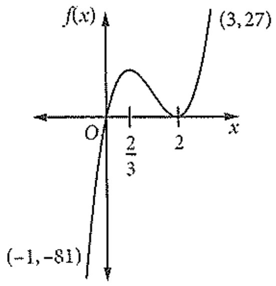


11



$y' = -(3x^2 - 4x + 1)$:
$\frac{1}{3}, 1\frac{23}{27}$ minimum,
$(1, 2)$ maximum

12 (a) $x = \frac{2}{3}, 2$ (b) $x < \frac{2}{3}, x > 2$ (c) $\frac{2}{3} < x < 2$
(d)



range = 108
greatest value = 27
least value = −81

13 $y' = 2ax + b$: $y' = 0$ where $2ax + b = 0$, $x = \frac{-b}{2a}$;
$y'' = 2a$ so the turning point is minimum if $a > 0$, maximum if $a < 0$

14 $\frac{dy}{dx} = -\frac{1}{x^2}$: $\frac{dy}{dx}$ is never zero, hence no stationary points, hence no turning points; $\frac{dy}{dx} < 0$ for all x in the domain

EXERCISE 14.3

1 (a) 6 (b) $6x + 4$ (c) -2 (d) $20x^3 + 12x$ (e) $-12x^2 + 4$ (f) 0

2 D

3 (a) correct (b) incorrect (c) correct (d) incorrect

4 (a) $\frac{-1}{4x\sqrt{x}}$ (b) $\frac{-1}{4(x-2)\sqrt{x-2}}$ (c) $\frac{x(2x^2+3)}{(x^2+1)^{\frac{3}{2}}}$
(d) $\frac{2}{x^3}$ (e) $\frac{2}{(x+1)^3}$ (f) $\frac{-6}{(x+3)^3}$
(g) $\frac{3(x^2+1)}{4x^2\sqrt{x}}$ (h) $\frac{15x^2+1}{4x\sqrt{x}}$ (i) $\frac{3x^2-18x+11}{4(x-1)^{\frac{3}{2}}(x+1)^3}$

5 real x 6 real x 7 (a) $x > -2$ (b) $x < -2$ (c) $(-2, 22)$

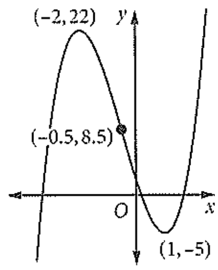
8 $x > -1$
$\frac{d^2y}{dx^2} \neq 0$ for x in the domain $\therefore$ no point of inflection

9 (a) $x > 0$ (b) $x < 0$
(c) $\frac{d^2y}{dx^2} \neq 0$ for x in the domain $\therefore$ no point of inflection

10 $\frac{d^2y}{dx^2} = \frac{6}{x^4}$; $\frac{d^2y}{dx^2} > 0$ for x in the domain $\therefore$ concave up

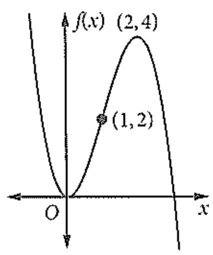
EXERCISE 14.4

1 $(-2, 22)$ maximum;
$(1, -5)$ minimum;

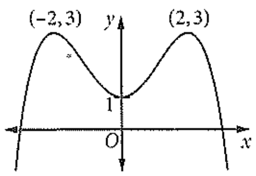


2 A

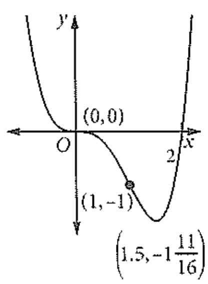
3 $(0, 0)$ minimum;
$(2, 4)$ maximum;
$(1, 2)$ inflection



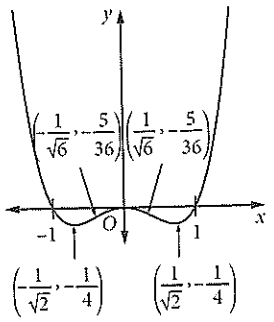
4 $(0, 1)$ min; $(2, 3)$ max; $(-2, 3)$ max;
greatest value of function is 3

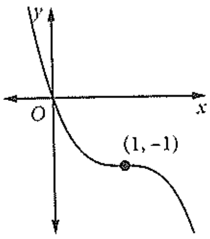


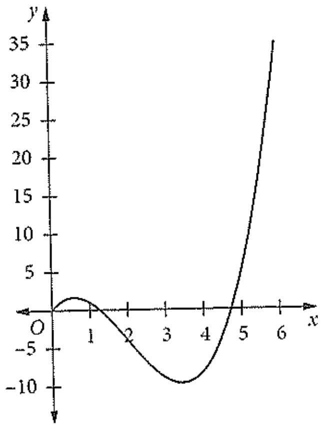
5 (a) $\left(1\frac{1}{2}, -1\frac{11}{16}\right)$ minimum
(b) $(0, 0)$ horizontal inflection, $(1, -1)$ inflection
(c)

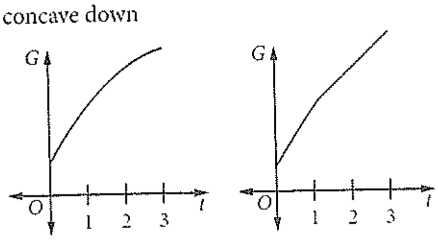


(d) $-1\frac{11}{16}$

6 (a) $x=2$ (b) $x>2$ (c) $x<2$
7 $(-1,-9)$, $(2,-48)$; $x<-1$, $x>2$
8 (a) $(-1,0)$, $(0,0)$, $(1,0)$
(b) $(0,0)$ max; $\left(\pm\frac{1}{\sqrt{2}},-\frac{1}{4}\right)$ min (c) $\left(\pm\frac{1}{\sqrt{6}},-\frac{5}{36}\right)$
(d)


(e) $-\frac{\sqrt{6}}{6}<x<\frac{\sqrt{6}}{6}$
9 (a) correct (b) correct (c) incorrect (d) correct
10 $\frac{dy}{dx}=-3(x-1)^2<0$ for all $x\neq 1$
$\frac{d^2y}{dx^2}=-6(x-1)=0$ where $x=1$;
concavity changes, so $(1,-1)$ is a point of inflection.
$y=-x(x^2-3x+3)$, $\Delta=9-12<0$,
$x^2-3x+3=0$ has no real roots;
the curve only cuts the x-axis at $(1,-1)$


11 (a) $(1,0)$ minimum
(b) $y''=12x^2$: $y''=0$ at $x=0$ but concavity does not change
$\therefore$ no point of inflection, curve is always concave up
(c) Global minimum is 0
12 (a) Greatest value of the function is 36 when $x=6$.
Least value of the function is $-4\left(1+\sqrt{2}\right)$ when $x=2+\sqrt{2}$.
(b)


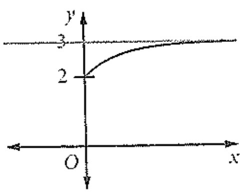
13 \$4.50
14 40 units
15 (a) 90 (b) \$17 420 (c) $P=374x-2.2x^2-400$ (d) 85, \$15 495
16 (a) decreasing (b) it is increasing at a decreasing rate
(c) concave down
(d)


(e) $f'(t)=3.2$, point of inflection
(f) it is increasing at a decreasing rate

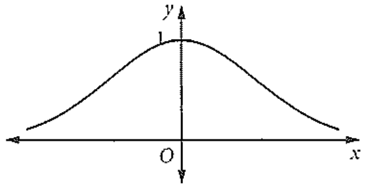
EXERCISE 14.5

1 (a) $l=80-x$ (b) $A(x)=x(80-x)=80x-x^2$
(c) 1600 m^2
2 B
3 (a) $l=160-2x$ (b) $A(x)=x(160-2x)$ (c) 3200 m^2
4 (a) $y=\frac{120-4x}{3}$ (b) $A(x)=\frac{x(120-4x)}{3}$
(c) $x=15$, $y=20$, $A=300\text{ m}^2$ (d) 396.75 m^2
5 (a) $A=\frac{1+x-x^2}{2}$ (b) $\frac{5}{8}$ units squared
6 $\frac{1600}{27}\text{ cm}^3$
7 (a) $y=12-4x$, $0<x<3$ (b) $V=12x^2-4x^3$ (c) 16 cm^2
8 $\frac{8000}{27}\text{ cm}^3$
9 (a) $h=9-2x$, $0<x<4.5$ (b) $V=x^2(9-2x)$ (c) $x=3$
10 (a) length $=2x$, height $=\frac{108-2x^2}{3x}$
(b) $V=\frac{2x\left(108-2x^2\right)}{3}$ (c) $144\sqrt{2}\text{ cm}^3$
11 (a) correct (b) correct (c) correct (d) correct
12 (a) $y=\sqrt{100-x^2}$ (b) $V=100x-x^3$ (c) $\frac{2000\sqrt{3}}{9}\text{ cm}^3$
13 (a) $r=\sqrt{36-x^2}$ (b) $V=\frac{\pi x\left(36-x^2\right)}{3}$ (c) $2\sqrt{3}$ cm
14 $28\frac{1}{8}\text{ cm}^2$
15 (a) $h=\frac{10-r^2}{r}$ (b) $V=\pi(10r-r^3)$ (c) $r=\frac{\sqrt{30}}{3}$
16 rectangle $3\frac{4}{7}$ cm $\times$ $10\frac{5}{7}$ cm; square sides $5\frac{5}{14}$ cm
17 (a) $R=(380-12(n-20))\times n=620n-12n^2$
(b) 26 passengers
18 (a) \$15 000 (b) 2812 or 2813
19 (a) \$24 000 (b) \$27 648
20 (a) $h^2+r^2=a^2$, $h=\sqrt{a^2-r^2}$, $V=\pi r^2\times 2h=2\pi r^2\sqrt{a^2-r^2}$
(b) $V'(r)=2\pi r\left(2a^2-3r^2\right)$;
$3r^2=2a^2$, $r=\frac{a\sqrt{6}}{3}$
$V''(r)=2\pi(2a^2-9r^2)$
$V''\left(\frac{a\sqrt{6}}{3}\right)=2\pi\left(2a^2-\frac{\cancel{9}\times 6a^2}{\cancel{9}}\right)<0$;
$r=\frac{a\sqrt{6}}{3}$ gives max volume

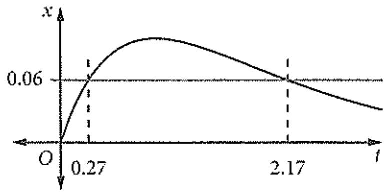
EXERCISE 14.6

1 $-e$
2 $\frac{dy}{dx}=\frac{(2-x)e^{-0.5x}}{2}$: $\left(2,\frac{2}{e}\right)$, maximum (a) $x>0$ (b) $x<2$
3 (a) $f(0)=2$, $f'(0)=1$ (b) $f'(x)=e^{-x}>0$ for all x (c) 3
(d)


4 (a) $f'(x) = -2xe^{-x^2}$ (b) (i) $x = 0$ (ii) $x < 0$ (iii) $x > 0$
(c)



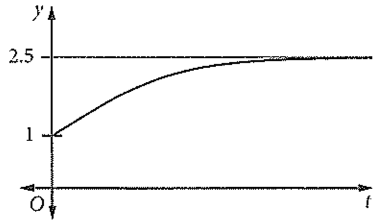
5 (a) 0.1 units after 0.91 hours
(b)



(c) 2.00 hours

6 (a) correct (b) incorrect (c) correct (d) correct

7



(a) $f'(t) = \dfrac{-5\times\left(-3e^{-t}\right)}{\left(2+3e^{-t}\right)^2} = \dfrac{15e^{-t}}{\left(2+3e^{-t}\right)^2} > 0$ because $e^{-t} > 0$ and $\left(2+3e^{-t}\right)^2 > 0$ for all t

(b) 2.5 (c) $1 \le f(t) < 2.5$

8 (a) $x = \dfrac{1}{\sqrt{2}}$ (b) $\sqrt{2}e^{-0.5} \approx 0.86$ units^2

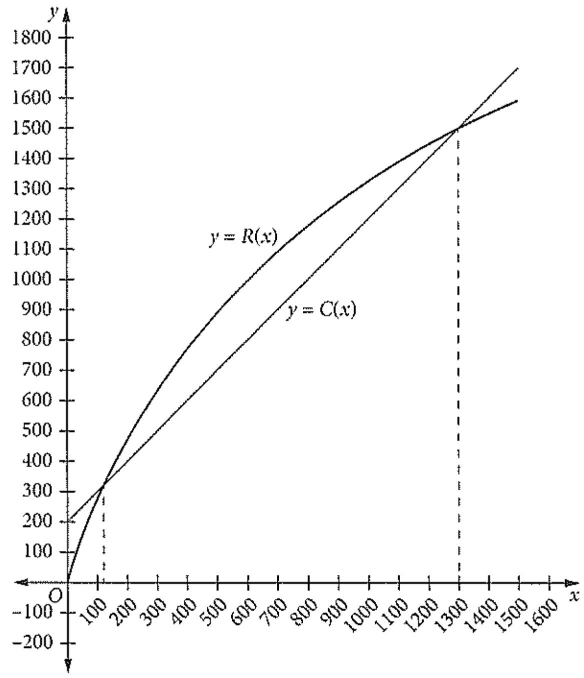
9 $\dfrac{dy}{dx} = \dfrac{1-\ln x}{x^2}$. Stationary points: $x = e$, $y = \dfrac{1}{e}$.
$\dfrac{d^2y}{dx^2} = \dfrac{2\ln x - 3}{x^3} < 0$ when $x = e$. Maximum at $\left(e, \dfrac{1}{e}\right)$.

10 $\dfrac{dy}{dx} = 2e^{2x} - 8e^{-2x}$. Stationary points: $x = \dfrac{\ln 2}{2}$, $y = 4$.
$\dfrac{d^2y}{dx^2} = 16 > 0$. Minimum value is 4 when $x = \dfrac{\ln 2}{2}$.

11 $\dfrac{dy}{dx} = 1 + \ln x$. Stationary points: $x = \dfrac{1}{e}$, $y = -\dfrac{1}{e}$. $\dfrac{d^2y}{dx^2} = e > 0$.
Minimum when $x = \dfrac{1}{e}$ is $-\dfrac{1}{e}$.

12 (a) Require $\sin x > 0$: $0 < x < \pi$.
(b) $\dfrac{dy}{dx} = \cot x$. Stationary points: $x = \dfrac{\pi}{2}$, $y = 0$.
$\dfrac{d^2y}{dx^2} = -\text{cosec}^2 x = -1 < 0$. Maximum when $x = \dfrac{\pi}{2}$ is 0.

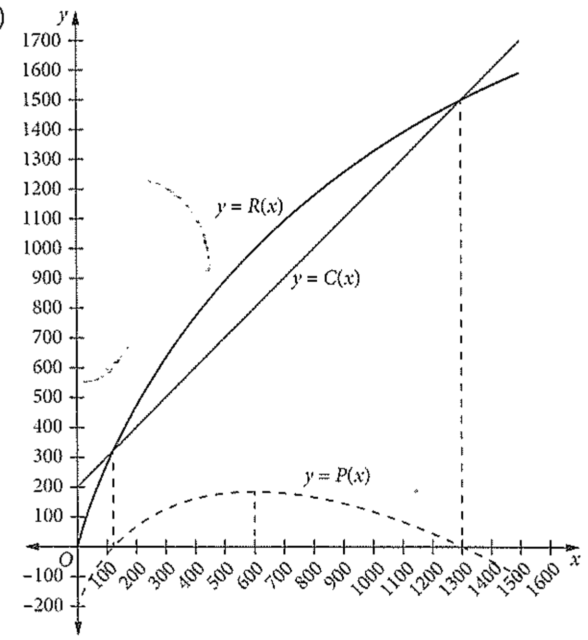
13 (a) Maximum profit is \$188.75
(b)



First breaks even when $x \approx 130$

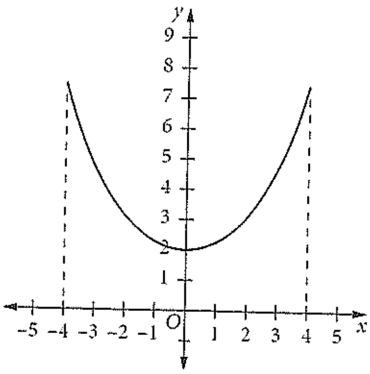
(c) A profit is made when the revenue > cost. This is between about 130 and 1300, or 13 batches of ten and 130 batches of ten.

(d)



$P(x) \ge 0$ for $130 \le x \le 1300$

14 (a) $a = 0.5,\ y = e^{\frac{x}{2}} + e^{-\frac{x}{2}}$



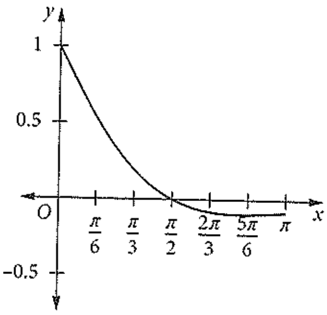
(b) Least height of function is 2 units so sag = 5.52 units.
(c) Angle of inclination at the ends is 74° 35′.

15 (a) $f(0) = 1,\ f\left(\frac{\pi}{2}\right) = 0, f(\pi) = -e^{-\pi}$

(b) $f'(x) = -e^{-x}(\sin x + \cos x)$

(c) $f'(0) = -1,\ f'\left(\frac{3\pi}{4}\right) = -e^{-\frac{3\pi}{4}}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = 0$

(d)



EXERCISE 14.7

1 $-\tan x$

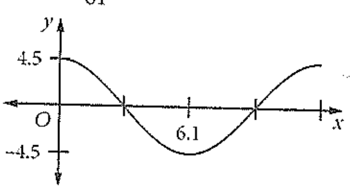
2 $4x - 2y + 2 - \pi = 0$

3 (a) $\cos x - \sin x$ (b) $-\sin x - \cos x$ (c) $\left(\frac{\pi}{4}, \sqrt{2}\right), \left(\frac{5\pi}{4}, -\sqrt{2}\right)$
(d) $\left(\frac{3\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 0\right)$ (e) $\sqrt{2}$

4 $y = 1 - x$

5 $\frac{dy}{dx} = 2\cos x + 2\sin x\cos x = 2\cos x(1 + \sin x)$; $\left(\frac{\pi}{2}, 3\right), \left(\frac{3\pi}{2}, -1\right)$, $\left(\frac{5\pi}{2}, 3\right), \left(\frac{7\pi}{2}, -1\right)$

6 (a) 4.5 m (b) 12.2 hours; $n = \frac{10\pi}{61}$
(c) $y = 4.5\cos\frac{10\pi t}{61}$



(d) 1.08 m

7 (a) $\frac{dy}{dx} = \cos x - \sin x$. $\frac{dy}{dx} = 0$ when $\tan x = 1$. $x = \frac{\pi}{4}, \frac{5\pi}{4}$.

(b) $x = \frac{\pi}{4}, y = \sqrt{2}$. $x = \frac{5\pi}{4}, y = -\sqrt{2}$. $\frac{d^2y}{dx^2} = -y$.

$x = \frac{\pi}{4}$: $\frac{d^2y}{dx^2} < 0$. Maximum at $\left(\frac{\pi}{4}, \sqrt{2}\right)$.

$x = \frac{5\pi}{4}$: $\frac{d^2y}{dx^2} > 0$. Minimum at $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$.

Greatest value of y is $\sqrt{2}$ at $x = \frac{\pi}{4}$ and the least value is $-\sqrt{2}$ at $x = \frac{5\pi}{4}$.

(c) $\frac{dy}{dx} > 0$ for $0 < x < \frac{\pi}{4}$ and $\frac{5\pi}{4} < x < 2\pi$.

(d) $\frac{d^2y}{dx^2} = 0$: $\tan x = -1$, $x = \frac{3\pi}{4}, \frac{7\pi}{4}$. Points of inflection at $\left(\frac{3\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 0\right)$.

8 $\frac{dy}{dx} = -\text{cosec}^2 x$. $P\left(\frac{\pi}{4}, 1\right)$: $\frac{dy}{dx} = -2$. Gradient of normal $= \frac{1}{2}$.
Equation of normal: $y - 1 = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$ or $4x - 8y + 8 - \pi = 0$

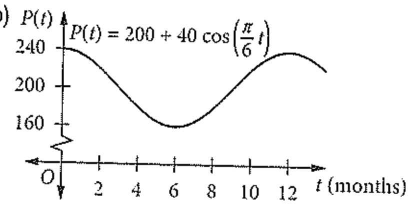
9 $\frac{dy}{dx} = e^{\text{cosec}\, x}(-\text{cosec}\, x \cot x)$. $x = \frac{\pi}{2}$: $\frac{dy}{dx} = 0, y = e$.
The equation of the tangent is $y = e$.

10 $y = 3\cos 4x$. $\frac{dy}{dx} = -12\sin 4x$. $\frac{d^2y}{dx^2} = -48\cos 4x$.

$\text{LHS} = \frac{d^2y}{dx^2} + 16y = -48\cos 4x + 16 \times 3\cos 4x$
$= -48\cos 4x + 48\cos 4x = 0 = \text{RHS}$

11 (a) $180 = 200 + 40\cos\left(\frac{\pi}{6}t\right)$. $\cos\left(\frac{\pi}{6}t\right) = -\frac{1}{2}$. $t = 4, 8$.
After 4 months and 8 months.

(b)



(c) $\frac{dP(t)}{dt} = -\frac{20\pi}{3}\sin\left(\frac{\pi}{6}t\right)$.

$\frac{dP(t)}{dt} > 0$: $\sin\left(\frac{\pi}{6}t\right) < 0$, $6 < t < 12$.

From the graph: between 6 and 12 months.
(d) The greatest population is 240 wallabies.
(e) From 160 to 240 wallabies.

EXERCISE 14.8

1 $x = \frac{t^3}{2} - 3t^2 + 5$
(a) $v = \frac{3t^2}{2} - 6t$ (b) $a = 3t - 6$
(c) $\frac{3t^2}{2} - 3t = 0$, $\frac{3t}{2}(t - 4) = 0$, $t = 0, 4$
(d) $t = 4$: $x = -11$, $v = 0$, $a = 6$

2 C

3 $x = 2t^3 - 6t^2 - 30t$.
(a) $v = 6t^2 - 12t - 30$, $a = 12t - 12$
(b) $t = 0$: $v = -30$, $a = -12$

(c) $v = 0$: $6t^2 - 12t - 30 = 0$, $6(t^2 - 2t - 5) = 0$,

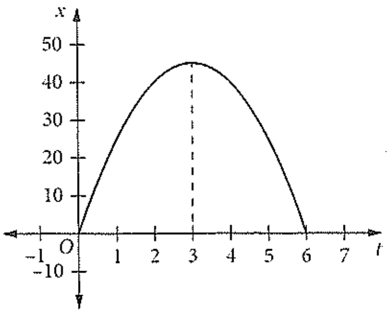
$t = \frac{2 \pm \sqrt{4+20}}{2} = 1 \pm \sqrt{6}$,

$t = 1 + \sqrt{6}$

$a = 12(1 + \sqrt{6}) - 12 = 12\sqrt{6}$

(d) $t^2 - 2t - 5 < 0$, $0 \le t < 1 + \sqrt{6}$

4 (a)



(b) $v = 30 - 10t$. (c) $v = 30\ \mathrm{m\,s^{-1}}$

(d) $v = 0$: $t = 3$ s, $x = 90 - 45 = 45$ m (e) 6 seconds

(f) $v = -30$, speed $= 30\ \mathrm{m\,s^{-1}}$ (g) $a = -10\ \mathrm{m\,s^{-2}}$

5 (a) $a = -5e^{-t}$. (b) $e^{-t} > 0$ for all t so $a < 0$ for all $t \ge 0$

(c) $v = 5e^{-t} > 0$ for all $t \ge 0$ as $e^{-t} > 0$ for all t. Hence $\frac{dx}{dt}$ is never zero so x cannot have any stationary points.

6 (a) $20.61\ \mathrm{km\,h^{-1}}$

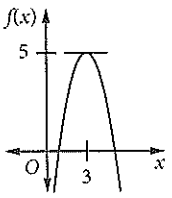
(b) Maximum velocity after 2.5 hours

CHAPTER REVIEW 14

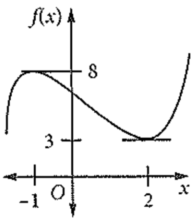
1 (a) $-0.5 < x < 2.5$ (b) $x < -0.5, x > 2.5$ (c) $x = 2.5$ (d) $x = -0.5$

2 (a) $x = -2, 1$ (b) $x < -2, x > 1$ (c) $-2 < x < 1$

3 (a)



(b)



4 (a) $(0,0)$ max; $\left(\frac{4}{9}, \frac{-32}{243}\right)$ min (b) $(-1, 5)$ max; $(3, -27)$ min

5 $x = \frac{15(6+\sqrt{3})}{11}$, $y = \frac{45(5-\sqrt{3})}{22}$

6 $3\frac{13}{17}$ m, $4\frac{4}{17}$ m

7 height $= y$ cm; $2x + 2y + \pi x = 50$, $y = \frac{50 - (\pi + 2)x}{2}$;

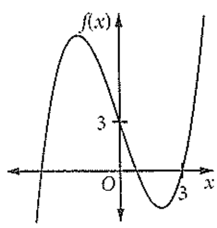
$\frac{100}{\pi + 4}$ by $\frac{50}{\pi + 4}$, $\frac{1250}{\pi + 4}\ \mathrm{cm^2}$

8 (a) $x = \pm 2$ (b) $-2 < x < 2$

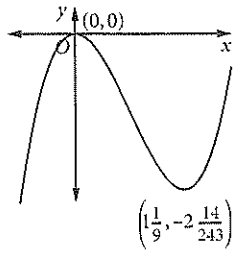
(c) $8\frac{1}{3}$, local max;

$-2\frac{1}{3}$, local min

(d)



9

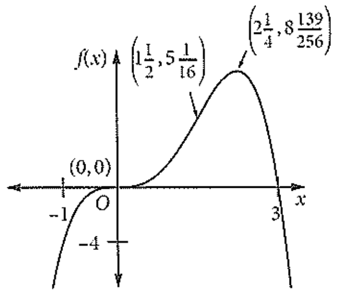


10 $\frac{5 - \sqrt{7}}{3}$ cm

11 $y' = 6x - 3x^2 = 3x(2 - x)$

At $x = 0$, $y = 0$, $y' = 0$, so the curve is horizontal where it cuts the y-axis at the origin and therefore crosses the y-axis at right angles.

12



$\left(2\frac{1}{4}, 8\frac{139}{256}\right)$ maximum turning point;

$(0,0)$ horizontal inflection, $\left(1\frac{1}{2}, 5\frac{1}{16}\right)$ inflection

13 (a) (i) $\frac{\sqrt{9+x^2}}{10\sqrt{2}}$ hours (ii) $\frac{4-x}{20}$ hours

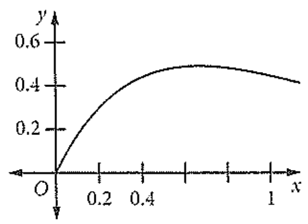
(b) $\frac{\sqrt{9+x^2}}{10\sqrt{2}} + \frac{4-x}{20}$

(c) $x = 3$ km, $t = 0.35$ h $= 21$ min

14 $60\ \mathrm{km\,h^{-1}}$

15 (a) 0.4905 at $x = \frac{2}{3}$

(b) $f(0) = 0$, $f(0.5) = 0.472$, $f(1) = 0.446$



16 $\frac{d}{dt}(\theta_0 e^{-kt}) = -k\theta_0 e^{-kt} = -k\theta$

17 (a) \$10 000 (b) \$3012

(c) (i) \$602.39 per year (ii) \$1000 per year

(d) 11.5 years

18 $\frac{dy}{dx} = 6 \sec 2x \tan 2x$. Stationary points: $x = 0, \frac{\pi}{2}$

$\frac{d^2y}{dx^2} = 12 \sec 2x(\tan^2 2x + \sec^2 2x)$

Minimum at $(0, 3)$, maximum at $\left(\frac{\pi}{2}, -3\right)$.