

Chapter 5

Exercise 5.01

- 1 **a** $12x^3 - 6x^2 + 7$ **b** 2
c $12x - 3$
- 2 $20x^4 + 18x$ **3** $6\pi t^2 - 6t$ **4** $f'(-2) = 101$
- 5 **a** $-5x^{-6}$ **b** $\frac{2}{3}x^{-\frac{1}{3}}$ **c** $-\frac{2}{x^3}$
d $\frac{1}{4\sqrt[4]{x^3}}$ **e** $\frac{20}{x^5}$
- 6 $\frac{1}{12}$
- 7 **a** $21(3x-1)^6$ **b** $3(2x-1)(x^2-x+2)^2$
c $\frac{7}{2\sqrt{7x-2}}$ **d** $-\frac{3}{(3x-2)^2}$
e $\frac{2x}{3\sqrt[3]{(x^2-3)^2}}$
- 8 **a** $3x^2 + 8x$ **b** $24x + 4$ **c** $12x^2 + 4$
d $4(x^2-1)(5x^2+3x-1)$ **e** $\frac{x^2(7x+6)}{\sqrt{x+1}}$
- 9 **a** $-\frac{13}{(x-5)^2}$ **b** $\frac{x^2(8x-21)}{(4x-7)^2}$
c $\frac{2(x^2-3x-3)}{(2x-3)^2}$ **d** $\frac{-6x+23}{(2x+9)^3}$
e $\frac{3x-7}{\sqrt{(2x-1)^3}}$
- 10 **a** -6 **b** 3
- 11 **a** $\frac{1}{14}$ **b** $\frac{1}{5}$
- 12 **a** $7x - y - 24 = 0$ **b** $51x - y - 72 = 0$
- 13 **a** $x - 4y - 3 = 0$ **b** $x + 3y - 6 = 0$
- 14 $x = 4$ **15** (2, 6), (-2, -10)
- 16 $3x + y + 8 = 0$ **17** (4, 2)
- 18 **a** $17x - y - 3 = 0$ **b** $x + 17y + 51 = 0$
- 19 **a** $6t$ **b** $-\frac{2}{(t-3)^2}$ **c** $\frac{2}{3\sqrt[3]{(2x+3)^2}}$
- 20 **a** **i** 10 kg s^{-1} **ii** 17 kg s^{-1}
b **i** 13 kg s^{-1} **ii** 123 kg s^{-1}
- 21 -2.18 Pa/m^3
- 22 **a** **i** 2 m **ii** 1.5 m **b** 2 s
c **i** 2 m s^{-1} **ii** 0 m s^{-1} **iii** -4 m s^{-1}

Exercise 5.02

- 1 **a** $7e^{7x}$ **b** $-e^{-x}$ **c** $6e^{6x-2}$
d $2xe^{x^2+1}$ **e** $(3x^2+5)e^{x^3+5x+7}$
f $5e^{5x}$ **g** $-2e^{-2x}$ **h** $10e^{10x}$
i $2e^{2x} + 1$ **j** $2x + 2 - e^{1-x}$
k $5(1+4e^{4x})(x+e^{4x})^4$ **l** $e^{2x}(2x+1)$
m $\frac{e^{3x}(3x-2)}{x^3}$ **n** $x^2e^{5x}(5x+3)$
o $\frac{4e^{2x+1}(x+2)}{(2x+5)^2}$
- 2 $3e$
- 3 **a** $3^x \ln 3$ **b** $10^x \ln 10$ **c** $3(2^{3x-4}) \ln 2$
- 4 5 **5** $x + y - 1 = 0$
- 6 **a** $3e^3$ **b** $-\frac{1}{3e^3}$
- 7 **a** $y = 2ex - e$ **b** $x + 2ey - 2e^2 - 1 = 0$
- 8 $x \ln 4 - y + 4 = 0$
- 9 **a** **i** 29 627 **ii** 35 826
b **i** 1044 people/year **ii** 1240 people/year
c **i** 1126 people/year **ii** 1361 people/year
- 10 **a** $55\,042 \text{ cm min}^{-1}$
b **i** $142.8 \text{ cm min}^{-1}$ **ii** 1087 cm min^{-1}
iii $177\,722\,205.4 \text{ cm min}^{-1}$
- 11 **a** 20 g **b** 7 g
c -0.091 g/year
d **i** -0.147 g/year **ii** -0.051 g/year
iii -0.0063 g/year
- 12 **a** $66\,079.4 \text{ cm}$ **b** $132\,158.8 \text{ cm s}^{-1}$

Exercise 5.03

- 1 **a** $1 + \frac{1}{x}$ **b** $-\frac{1}{x}$ **c** $\frac{3}{3x+1}$
d $\frac{2x}{x^2-4}$ **e** $\frac{15x^2+3}{5x^3+3x-9}$
f $\frac{10x^2+2x+5}{5x+1}$ **g** $6x+5+\frac{1}{x}$
h $\frac{8}{8x-9}$ **i** $\frac{6x+5}{(x+2)(3x-1)}$
j $\frac{-30}{(4x+1)(2x-7)}$ **k** $\frac{5}{x}(1+\ln x)^4$
l $9\left(\frac{1}{x}-1\right)(\ln x-x)^8$ **m** $\frac{4}{x}(\ln x)^3$

$$\mathbf{n} \quad 6\left(2x + \frac{1}{x}\right)(x^2 + \ln x)^5$$

$$\mathbf{o} \quad 1 + \ln x$$

$$\mathbf{q} \quad \frac{2x \ln x + 2x + 1}{x}$$

$$\mathbf{r} \quad 3x^2 \ln(x+1) + \frac{x^3}{x+1}$$

$$\mathbf{t} \quad \frac{x-2-x \ln x}{x(x-2)^2}$$

$$\mathbf{v} \quad e^x \left(\frac{1}{x} + \ln x \right)$$

$$\mathbf{p} \quad \frac{1 - \ln x}{x^2}$$

$$\mathbf{s} \quad \frac{1}{x \ln x}$$

$$\mathbf{u} \quad \frac{e^{2x}(2x \ln x - 1)}{x(\ln x)^2}$$

$$\mathbf{w} \quad \frac{10 \ln x}{x}$$

$$2 \quad f'(1) = -\frac{1}{2}$$

$$3 \quad \frac{1}{x \ln 10}$$

$$4 \quad x - 2y - 2 + 2 \ln 2 = 0$$

$$5 \quad x - y - 2 = 0$$

$$6 \quad -\frac{2}{5}$$

$$7 \quad 5x + y - \ln 5 - 25 = 0$$

$$8 \quad 5x - 19y + 19 \ln 19 - 15 = 0$$

$$9 \quad \frac{2}{(2x+5) \ln 3}$$

$$10 \quad (2 \ln 2)x + y - 1 - 4 \ln 2 = 0$$

$$11 \quad \mathbf{a} \quad 20\,000$$

$$\mathbf{b} \quad \mathbf{i} \quad 10.6 \text{ years}$$

$$\mathbf{ii} \quad 43.6 \text{ years}$$

$$\mathbf{c} \quad P = 20\,000 e^{0.021t}$$

$$\mathbf{d} \quad 452 \text{ kangaroos/year}$$

$$\mathbf{e} \quad \mathbf{i} \quad 447 \text{ kangaroos/year}$$

$$\mathbf{ii} \quad 466 \text{ kangaroos/year}$$

$$\mathbf{iii} \quad 518 \text{ kangaroos/year}$$

Exercise 5.04

$$1 \quad \mathbf{a} \quad 4 \cos 4x$$

$$\mathbf{b} \quad -3 \sin 3x$$

$$\mathbf{c} \quad 5 \sec^2 5x$$

$$\mathbf{d} \quad 3 \sec^2(3x+1)$$

$$\mathbf{e} \quad \sin(-x)$$

$$\mathbf{f} \quad 3 \cos x$$

$$\mathbf{g} \quad -20 \sin(5x-3)$$

$$\mathbf{h} \quad -6x^2 \sin(x^3)$$

$$\mathbf{i} \quad 14x \sec^2(x^2+5)$$

$$\mathbf{j} \quad 3 \cos 3x - 8 \sin 8x$$

$$\mathbf{k} \quad \sec^2(\pi+x) + 2x$$

$$\mathbf{l} \quad x \sec^2 x + \tan x$$

$$\mathbf{m} \quad 3 \sin 2x \sec^2 3x + 2 \tan 3x \cos 2x$$

$$\mathbf{n} \quad \frac{x \cos x - \sin x}{2x^2}$$

$$\mathbf{o} \quad \frac{3 \sin 5x - 5(3x+4) \cos 5x}{\sin^2 5x}$$

$$\mathbf{p} \quad 9(2+7 \sec^2 7x)(2x+\tan 7x)^8$$

$$\mathbf{q} \quad 2 \sin x \cos x = \sin 2x \quad \mathbf{r} \quad -45 \sin 5x \cos^2 5x$$

$$\mathbf{s} \quad e^x + 2 \sin 2x \quad \mathbf{t} \quad -\frac{1}{x} \cos(1 - \ln x)$$

$$\mathbf{u} \quad (e^x + 1) \cos(e^x + x) \quad \mathbf{v} \quad \frac{\cos x}{\sin x} = \cot x$$

$$\mathbf{w} \quad e^{3x}(3 \cos 2x - 2 \sin 2x)$$

$$\mathbf{x} \quad \frac{e^{2x}(2 \tan 7x - 7 \sec^2 7x)}{\tan^2 7x}$$

$$2 \quad 12$$

$$3 \quad 6\sqrt{3}x - 12y + 6 - \pi\sqrt{3} = 0$$

$$4 \quad -\frac{\sin x}{\cos x} = -\tan x \quad 5 \quad -\frac{2}{3\sqrt{3}} = -\frac{2\sqrt{3}}{9}$$

$$6 \quad \sec^2 x e^{\tan x}$$

$$7 \quad 8x + 24\sqrt{2}y - 72 - \pi = 0$$

$$8 \quad \text{Proof (see worked solutions).}$$

$$9 \quad \mathbf{a} \quad \frac{\pi}{180} \sec^2 x^\circ \quad \mathbf{b} \quad -\frac{\pi}{60} \sin x^\circ \quad \mathbf{c} \quad \frac{\pi}{900} \cos x^\circ$$

$$10 \quad \sin^3 x (4 \cos^2 x - \sin^2 x)$$

$$11 \quad \mathbf{a} \quad 750 \quad \mathbf{b} \quad 525 \quad \mathbf{c} \quad 975$$

$$\mathbf{d} \quad 2.6, 6.4, 11.6, 15.4 \dots \text{ days}$$

$$\mathbf{e} \quad \mathbf{i} \quad -136 \text{ fish/day} \quad \mathbf{ii} \quad 155 \text{ fish/day}$$

$$\mathbf{iii} \quad -101 \text{ fish/day} \quad \mathbf{iv} \quad 0 \text{ fish/day}$$

$$\mathbf{f} \quad 0.23, 4.27, 9.23, 13.27, \dots \text{ days}$$

$$12 \quad \mathbf{a} \quad \mathbf{i} \quad 9 \text{ m} \quad \mathbf{ii} \quad 13 \text{ m}$$

$$\mathbf{b} \quad 0.2, 5.8, 12.2, 17.8, \dots \text{ h}$$

$$\mathbf{c} \quad \mathbf{i} \quad 0 \text{ m h}^{-1} \quad \mathbf{ii} \quad 3.6 \text{ m h}^{-1} \quad \mathbf{iii} \quad 4.2 \text{ m h}^{-1}$$

$$\mathbf{d} \quad 1.5, 10.5, 13.5, 22.5, \dots \text{ h}$$

Exercise 5.05

$$1 \quad 7x^6 - 10x^4 + 4x^3 - 1; 42x^5 - 40x^3 + 12x^2; 210x^4 - 120x^2 + 24x; 840x^3 - 240x + 24$$

$$2 \quad 72x^7$$

$$3 \quad f'(x) = 10x^4 - 3x^2, f''(x) = 40x^3 - 6x$$

$$4 \quad f'(1) = 11, f''(-2) = 168$$

$$5 \quad 7x^6 - 12x^5 + 16x^3; 42x^5 - 60x^4 + 48x^2; 210x^4 - 240x^3 + 96x$$

$$6 \quad \frac{dy}{dx} = 4x - 3, \frac{d^2y}{dx^2} = 4$$

$$7 \quad f'(-1) = -16, f''(2) = 40 \quad 8 \quad -4x^{-5}; 20x^{-6}$$

$$9 \quad -\frac{1}{32}$$

$$10 \quad 26$$

$$11 \quad x = \frac{7}{18}$$

$$12 \quad x > \frac{1}{3}$$

$$13 \quad 20(4x-3)^4; 320(4x-3)^3$$

$$14 \quad f'(x) = -\frac{1}{2\sqrt{2-x}}; f''(x) = -\frac{1}{4\sqrt{(2-x)^3}}$$

$$15 \quad f'(x) = -\frac{16}{(3x-1)^2}; f''(x) = \frac{96}{(3x-1)^3}$$

$$16 \quad \frac{d^2v}{dt^2} = 24t + 16 \quad 17 \quad b = \frac{2}{3}$$

$$18 \quad 196 \quad 19 \quad b = -2.7$$

$$20 \quad \frac{dy}{dx} = 4e^{4x} - 4e^{-4x}$$

$$\frac{d^2y}{dx^2} = 16e^{4x} + 16e^{-4x} = 16y$$

21, 22 Proofs (see worked solutions).

$$23 \quad n = -15$$

$$24 \quad y = 2 \cos 5x; \frac{dy}{dx} = -10 \sin 5x$$

$$\frac{d^2y}{dx^2} = -50 \cos 5x = -25y$$

$$25 \quad f(x) = -2 \sin x; f'(x) = -2 \cos x$$

$$f''(x) = 2 \sin x = -f(x)$$

$$26 \quad y = 2 \sin 3x - 5 \cos 3x; \frac{dy}{dx} = 6 \cos 3x + 15 \sin 3x$$

$$\frac{d^2y}{dx^2} = -18 \sin 3x + 45 \cos 3x = -9y$$

$$27 \quad a = -7, b = -24 \quad 28 \quad f''(2) = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

$$29 \quad \mathbf{a} \quad 8 \text{ m} \quad \mathbf{b} \quad 38 \text{ m} \quad \mathbf{c} \quad 31 \text{ m s}^{-1}$$

$$\mathbf{d} \quad 26 \text{ m s}^{-2}$$

$$30 \quad \mathbf{a} \quad 4 \text{ cm}$$

$$\mathbf{b} \quad \text{Maximum } 20 \text{ cm, minimum } 4 \text{ cm}$$

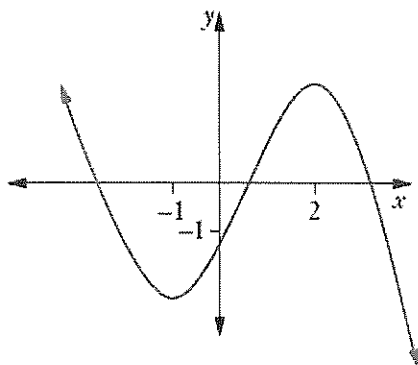
$$\mathbf{c} \quad \mathbf{i} \quad 0 \text{ cm s}^{-1} \quad \mathbf{ii} \quad 25.1 \text{ cm s}^{-1}$$

$$\mathbf{d} \quad \mathbf{i} \quad -8\pi^2 \text{ or } -79 \text{ cm s}^{-2} \quad \mathbf{ii} \quad 8\pi^2 \text{ or } 79 \text{ cm s}^{-2}$$

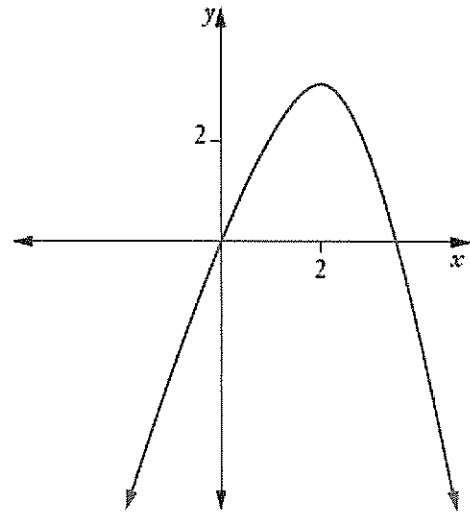
$$\mathbf{iii} \quad 0 \text{ cm s}^{-2} \quad \mathbf{e} \quad \frac{d^2h}{dt^2} = -\pi^2(h - 12)$$

Exercise 5.06

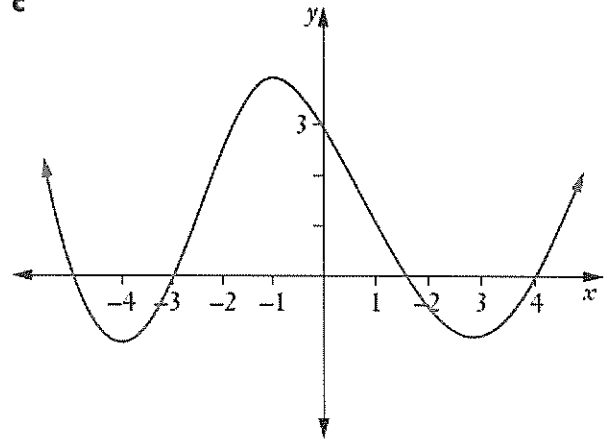
1 a



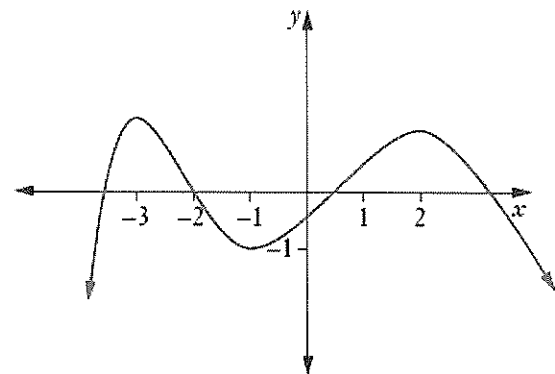
b



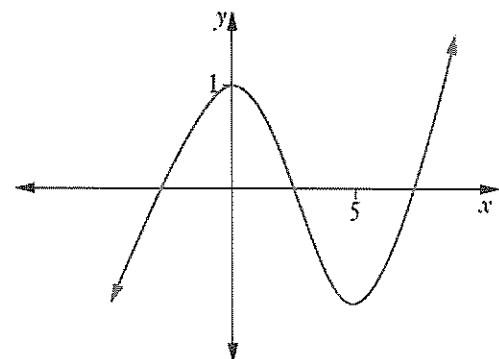
c



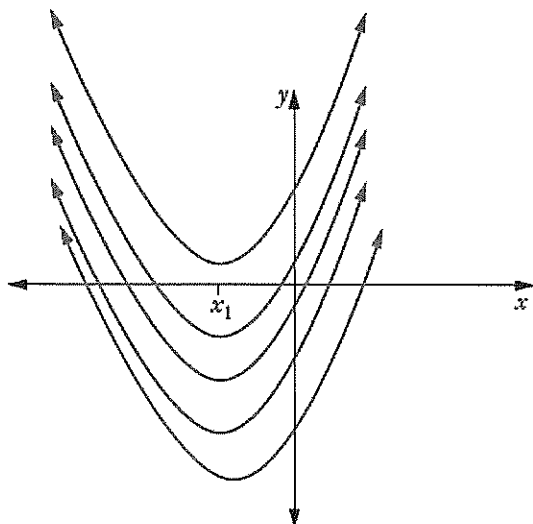
d



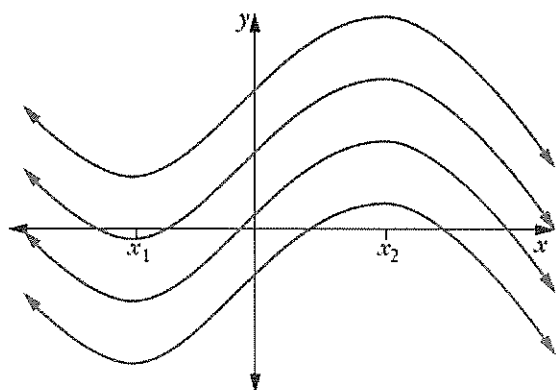
e



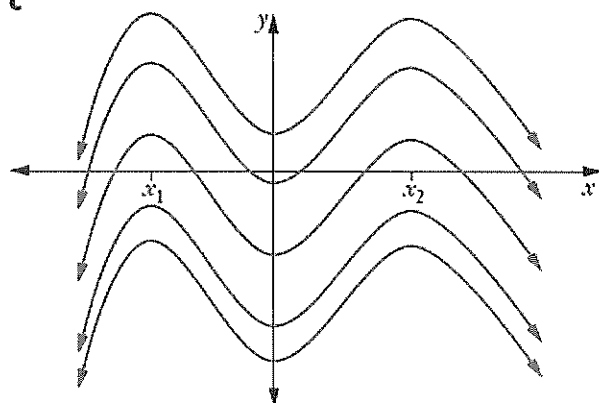
2 a



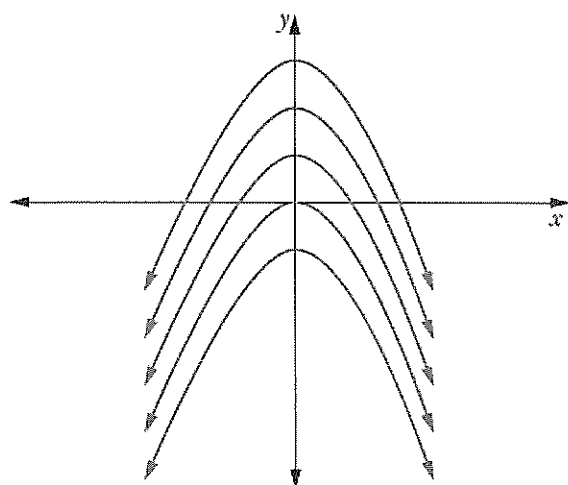
b



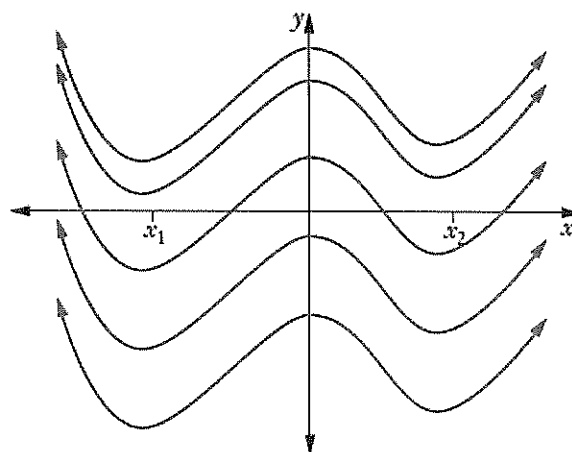
c



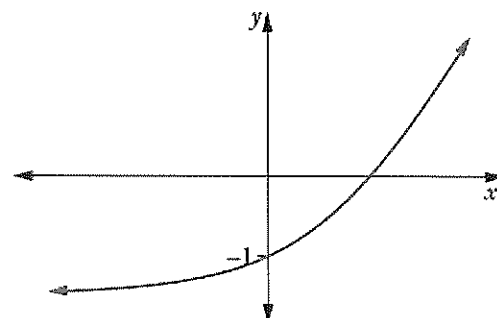
d



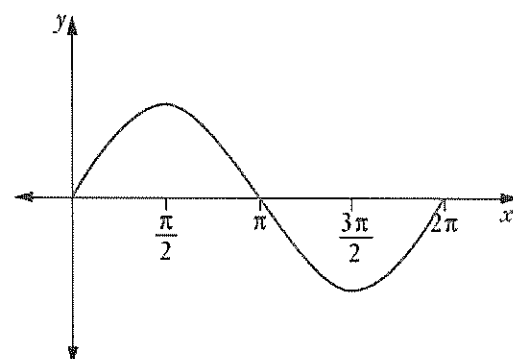
e



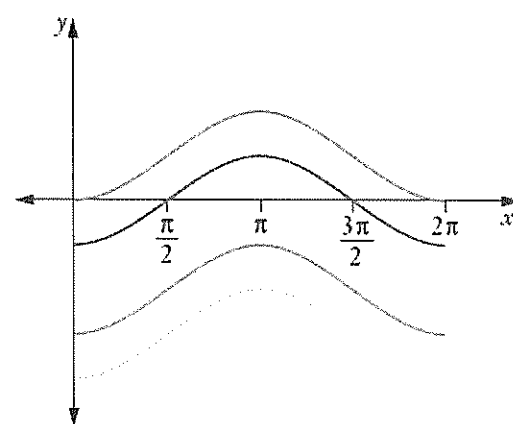
3



4



5



Exercise 5 07

1 a $x^2 - 3x + C$

b $\frac{x^3}{3} + 4x^2 + x + C$

c $\frac{x^6}{6} - x^4 + C$

d $\frac{(x-1)^3}{3} + C$

$$\mathbf{e} \quad 6x + C \quad \mathbf{f} \quad \frac{(3x+2)^6}{18} + C$$

$$\mathbf{g} \quad \frac{4(2x-7)^5}{5} + C$$

$$\mathbf{2} \quad \mathbf{a} \quad f(x) = 2x^3 - \frac{x^2}{2} + C$$

$$\mathbf{b} \quad f(x) = \frac{x^5}{5} - x^3 + 7x + C$$

$$\mathbf{c} \quad f(x) = \frac{x^2}{2} - 2x + C$$

$$\mathbf{d} \quad f(x) = \frac{x^3}{3} - x^2 - 3x + C \quad \mathbf{e} \quad f(x) = \frac{2x^{\frac{3}{2}}}{3} + C$$

$$\mathbf{3} \quad \mathbf{a} \quad y = x^5 - 9x + C \quad \mathbf{b} \quad y = -\frac{x^{-3}}{3} + 2x^{-1} + C$$

$$\mathbf{c} \quad y = \frac{x^4}{20} - \frac{x^3}{3} + C \quad \mathbf{d} \quad y = -\frac{2}{x} + C$$

$$\mathbf{e} \quad y = \frac{x^4}{4} - \frac{x^2}{3} + x + C$$

$$\mathbf{4} \quad \mathbf{a} \quad \frac{2\sqrt{x^3}}{3} + C \quad \mathbf{b} \quad -\frac{x^{-2}}{2} + C$$

$$\mathbf{c} \quad -\frac{1}{7x^7} + C \quad \mathbf{d} \quad 2x^{\frac{1}{2}} + 6x^{\frac{1}{3}} + C$$

$$\mathbf{e} \quad -\frac{x^{-6}}{6} + 2x^{-1} + C$$

$$\mathbf{5} \quad \mathbf{a} \quad \frac{(x^2+5)^5}{5} + C \quad \mathbf{b} \quad \frac{(x^3-1)^{10}}{10} + C$$

$$\mathbf{c} \quad \frac{(2x^2+3)^4}{2} + C \quad \mathbf{d} \quad \frac{3(x^5+1)^7}{7} + C$$

$$\mathbf{e} \quad \frac{(x^2-4)^8}{16} + C \quad \mathbf{f} \quad \frac{(2x^6-7)^9}{108} + C$$

$$\mathbf{g} \quad \frac{(x^2-x+3)^5}{5} + C \quad \mathbf{h} \quad \frac{(x^3+2x^2-7x)^{11}}{11} + C$$

$$\mathbf{i} \quad \frac{(x^2-6x-1)^6}{12} + C$$

$$\mathbf{6} \quad y = \frac{x^4}{4} - x^3 + 5x - \frac{1}{4}$$

$$\mathbf{7} \quad f(x) = 2x^2 - 7x + 11 \quad \mathbf{8} \quad f(1) = 8$$

$$\mathbf{9} \quad y = 2x - 3x^2 + 19 \quad \mathbf{10} \quad x = 16\frac{1}{3}$$

$$\mathbf{11} \quad y = 4x^2 - 8x + 7 \quad \mathbf{12} \quad y = 2x^3 + 3x^2 + x - 2$$

$$\mathbf{13} \quad f(x) = x^3 - x^2 - x + 5 \quad \mathbf{14} \quad f(2) = 20.5$$

$$\mathbf{15} \quad y = \frac{4x^3}{3} - 15x - 14\frac{1}{3}$$

$$\mathbf{16} \quad y = \frac{x^3}{3} - 2x^2 + 3x - 4\frac{2}{3}$$

$$\mathbf{17} \quad f(x) = x^4 - x^3 + 2x^2 + 4x - 2$$

$$\mathbf{18} \quad y = 3x^2 + 8x + 8$$

$$\mathbf{19} \quad f(-2) = 77$$

$$\mathbf{20} \quad x = 3t^2 - 5t - 2$$

$$\mathbf{21} \quad x = 2t^4 - 2t^3 + 3t^2 - 8t - 3$$

Exercise 5.08

$$\mathbf{1} \quad \mathbf{a} \quad -\cos x + C \quad \mathbf{b} \quad \tan x + C \quad \mathbf{c} \quad \sin x + C$$

$$\mathbf{d} \quad \frac{1}{7} \tan 7x + C \quad \mathbf{e} \quad -\frac{1}{2} \cos(2x - \pi) + C$$

$$\mathbf{2} \quad \mathbf{a} \quad e^x + C \quad \mathbf{b} \quad \frac{1}{6} e^{6x} + C \quad \mathbf{c} \quad \ln|x| + C$$

$$\mathbf{d} \quad \ln|3x-1| + C \quad \mathbf{e} \quad \frac{1}{2} \ln|x^2+5| + C$$

$$\mathbf{3} \quad \mathbf{a} \quad e^x + 5x + C \quad \mathbf{b} \quad \sin x + 2x^2 + C$$

$$\mathbf{c} \quad \frac{x^2}{2} + \ln|x| + C$$

$$\mathbf{d} \quad 2x^4 - x^3 + 3x^2 - 3x + \ln|x| + C$$

$$\mathbf{e} \quad -\frac{1}{5} \cos 5x - \frac{1}{9} \tan 9x + C$$

$$\mathbf{4} \quad y = \sin x - 5 \quad \mathbf{5} \quad f(x) = 5 \ln|x| + 3$$

$$\mathbf{6} \quad y = 2 \sin 2x + \sqrt{3}$$

$$\mathbf{7} \quad f(x) = 3e^{3x} - 8e^6x + 14e^6$$

$$\mathbf{8} \quad \mathbf{a} \quad P = 25\,000e^{0.054t} + 10\,000$$

$$\mathbf{b} \quad 52\,900$$

$$\mathbf{9} \quad x = e^{3t} + 4$$

$$\mathbf{10} \quad \mathbf{a} \quad \frac{dx}{dt} = 3 \cos 3t$$

$$\mathbf{b} \quad -0.3 \text{ cm} \quad \mathbf{c} \quad 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \dots \text{ s}$$

Exercise 5.09

$$\mathbf{1} \quad \mathbf{a} \quad \frac{dy}{dx} \times \frac{dx}{dy} = 4 \times \frac{1}{4} = 1$$

$$\mathbf{b} \quad \frac{dy}{dx} \times \frac{dx}{dy} = 3x^2 \times \frac{1}{3x^2} = 1$$

$$\mathbf{c} \quad \frac{dy}{dx} \times \frac{dx}{dy} = e^x \times \frac{1}{e^x} = 1$$

$$\mathbf{d} \quad \frac{dy}{dx} \times \frac{dx}{dy} = \frac{1}{x} \times x = 1$$

$$\mathbf{e} \quad \frac{dy}{dx} \times \frac{dx}{dy} = 7x^6 \times \frac{1}{7x^6} = 1$$

2 a $\frac{1}{x}$ b e^x c $2x$

d $\frac{1}{7\sqrt[7]{(x+1)^6}}$ e $\frac{1}{3\sqrt[3]{x^2}}$

3 a $\frac{1}{3}$ b $\frac{1}{2}$ c $\frac{1}{3}$

d 4 e $-\frac{27}{4}$

4 a $\frac{1}{12}\sqrt[3]{\left(\frac{4}{x}\right)^2}$

b i $\frac{1}{12}$ ii -12

5 a $f^{-1}(x) = \sqrt{x-1}$ b $\frac{1}{2\sqrt{x-1}}$

c $\frac{1}{4}$

6 a $ye^y(2+y)$ b $3(\sin 2y + 2y \cos 2y)$

c $(10y-3)(2y-3)^3$

d $\frac{17}{(2y+5)^2}$ e $\frac{y+2-y \ln y}{y(y+2)^2}$

7 a 47 b 1 c 0

d 1 e $\frac{16}{\pi^3}$

Exercise 5.10

1 a $-\frac{1}{\sqrt{1-x^2}}$ b $\frac{2}{\sqrt{1-x^2}}$ c $\frac{1}{1+x^2}$

d $-\frac{3}{\sqrt{1-9x^2}}$ e $\frac{8}{\sqrt{1-4x^2}}$ f $\frac{2x}{\sqrt{1-x^4}}$

g $\frac{2}{1+(2x-1)^2} = \frac{1}{2x^2-2x+1}$

h $-\frac{40}{\sqrt{1-64x^2}}$ i $-\frac{1}{\sqrt{9-x^2}}$ j $\frac{2}{4+x^2}$

2 a i -1 ii 1

b i $1\frac{3}{5}$ ii $-\frac{5}{8}$

c i $\frac{\pi^2}{6\sqrt{3}}$ ii $-\frac{6\sqrt{3}}{\pi^2}$

d i $-\frac{1}{3}$ ii 3

e i $\frac{1}{5}$ ii -5

3 $y = 2x$

4 $40x + 100y - 25\pi - 8 = 0$

5 a $\frac{3}{\sqrt{36-x^2}}$ b $-\frac{3}{2\sqrt{x(1-x^2)}}$

c $-\frac{1}{\sqrt{49-x^2}}$ d $\frac{15}{\sqrt{-9x^2-12x-3}}$

e $-\frac{x}{\sqrt{1-x^2}} + \cos^{-1} x$ f $\frac{5}{1+x^2}(\tan^{-1} x + 1)^4$

6 a -1 b 1

c $\frac{1}{x\sqrt{1-(\ln x)^2}}$ d $\frac{e^x}{1+e^{2x}}$

e $\frac{1}{\sin^{-1} x \cdot \sqrt{1-x^2}}$ f $-\frac{1}{(1+x^2)(\tan^{-1} x)^2}$

g $-\frac{1}{\sqrt{1-x^2}[1+(\cos^{-1} x+1)^2]}$ h $-\frac{1}{1+x^2}$

i $\frac{1}{\sqrt{-x^2-4x}}$ j $-\frac{e^{\cos^{-1} x}}{\sqrt{1-x^2}}$

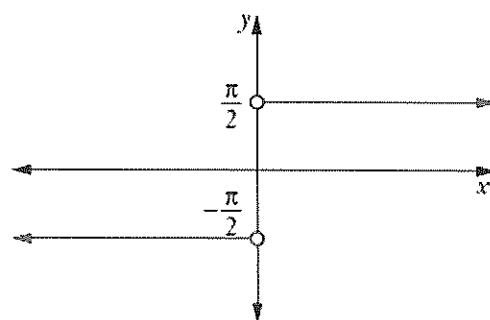
7 $\frac{d}{dx}(\sin^{-1} x + \cos^{-1} x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$

8 a $-\frac{x}{\sqrt{(9-x^2)^3}}$ b $\frac{-(1+2x \tan^{-1} x)}{[(1+x^2)(\tan^{-1} x)]^2}$

9 $12x - 6\sqrt{3}y - \sqrt{3}\pi + 6 = 0$

10 a 0

b



11 a $-\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$

b $\frac{1}{(1+x^2)\tan^{-1} x}$

c $\frac{1}{x[1+(\ln x)^2]}$

d $-\frac{1}{\sqrt{1-x^2}}$

e $\frac{e^{\tan^{-1} x}}{1+x^2}$

12 a $\sin \theta = \frac{h}{6}, \therefore \theta = \sin^{-1} \frac{h}{6}$

b -0.009 radians per second

13 a $2t$ m b $\theta = \tan^{-1} \left(\frac{t}{50} \right)$

c i 0.02 radians s^{-1} ii 0.0082 radians s^{-1}

14 a $\theta = \tan^{-1} \left(\frac{5+0.5t}{8} \right)$

b i 0.033 radians min^{-1} ii 0.014 radians min^{-1}
iii 0.0031 radians min^{-1}

Test yourself 5

1 D 2 B 3 A 4 C

5 a $5e^{5x}$ b $-2e^{1-x}$ c $\frac{1}{x}$

d $\frac{4}{4x+5}$ e $e^x(x+1)$ f $\frac{1-\ln x}{x^2}$

g $10e^x(e^x+1)^9$

6 a $-\sin x$ b $2 \cos x$ c $\sec^2 x$

d $x \cos x + \sin x$ e $\frac{x \sec^2 x - \tan x}{x^2}$

f $-3 \sin 3x$ g $5 \sec^2 5x$

7 $3x - y + 3 = 0$

8 $12x + 4\sqrt{2}y - 4 - 3\pi = 0$

9 $\frac{d^2x}{dt^2} = -4 \cos 2t$

10 a $\frac{1}{\sqrt{1-x^2}}$ b $\frac{3}{1+9x^2}$

c $-\frac{10}{\sqrt{1-25x^2}}$

11 $-\frac{e^2}{e^2+1}$

12 a $2x^5 - x^4 + 3x^2 - 3x + C$

b $\frac{1}{5}e^{5x} + C$

c $\frac{1}{9} \tan 9x + C$

d $\ln|x+5| + C$

e $\frac{1}{2} \sin 2x + C$

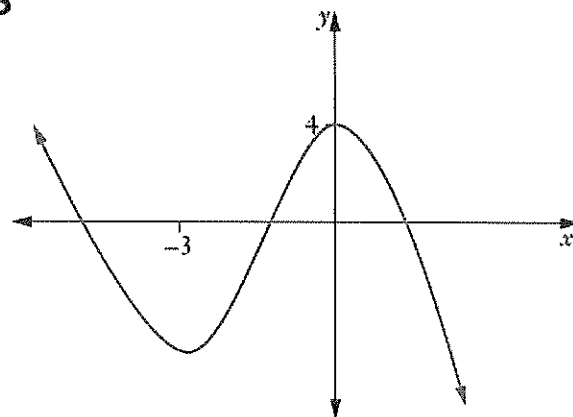
f $-4 \cos \frac{x}{4} + C$

13 $-3\sqrt{3}$

14 $y = 2x^3 + 6x^2 - 5x - 33$

15 $\frac{1}{5\sqrt[5]{(x-3)^4}}$

16



17 $2x + y - \ln 2 - 4 = 0$

18 $4x + 8y - 8 - \pi = 0$

19 a $\frac{1}{\sqrt{1-x^2}}$ b $-\frac{1}{\sqrt{25-x^2}}$ c $\frac{1}{1+x^2}$

d $\frac{4}{\sqrt{1-16x^2}}$ e $\frac{2}{4+x^2}$

20 a $40x(5x^2+7)^3$

b $4(16x-3)(2x-3)^6$

c $\frac{23}{(3x+4)^2}$ d $2x^2e^x(x+3)$

e $\frac{3(x+1)\sec^2 3x - \tan 3x}{(x+1)^2}$

21 a 0

b $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}; \frac{d}{dx} \left(\frac{\pi}{2} \right) = 0$

22 $f(x) = \frac{5x^3}{2} + 6x^2 - 49x + 59$

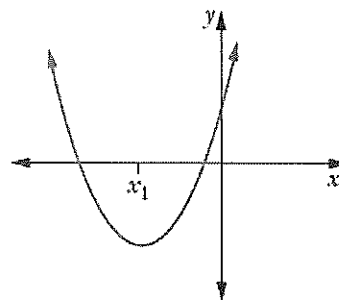
23 a -8 b 26 c -90

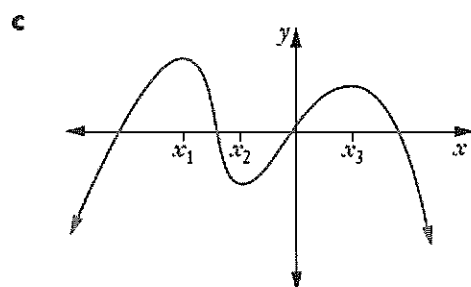
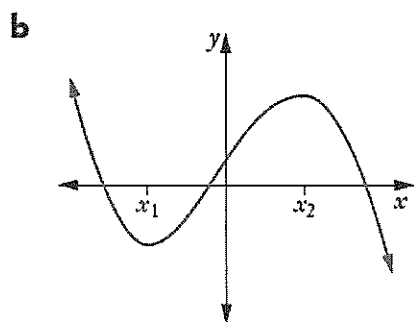
24 a $f^{-1}(x) = x^2 - 1$ b $P = (3, 8)$

c $6x - y - 10 = 0$

25 $\frac{x}{1+x^2} + \tan^{-1} x$

26 a





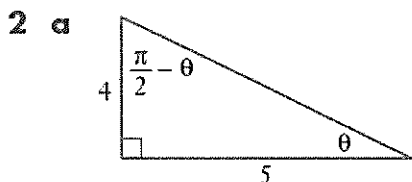
27 $f(x) = 2x^3 - 3x^2 - 31x + 68$

28 $4x - 6\sqrt{3}y + \sqrt{3}\pi - 6 = 0$

29 a $\frac{(3x^4 - 5)^7}{84} + C$ **b** $\frac{3(x^2 + 1)^{10}}{20} + C$

Challenge exercise 5

1 $2e$



b 0

c $\theta + \frac{\pi}{2} - \theta$

3 $\frac{20x^2 - 120x - 1}{(4x^2 + 1)^4}; \frac{-24(20x^3 - 140x^2 - 3x + 5)}{(4x^2 + 1)^5}$

4 a $e^{x^2} + C$ **b** $-\frac{1}{3} \cos(x^3) + C$

5 $(2x \cos 2x + \sin 2x)e^{x \sin 2x}$

6 $y = \frac{x^3}{3} - x^2 - 15x - 1$

7 a $\frac{2x}{\sqrt{1-x^4}}$ **b** $\frac{e^x}{1+e^{2x}}$

c $\frac{\cos x - \sin x}{\sin x + \cos x}$

8 $25\frac{5}{6}$

9 $\frac{1 + \ln x - x \ln x}{e^{2x}}$

10 a $\theta = \tan^{-1}(4t)$

b $3^\circ 32'$

11 a $x = y + e^y$

b $P = (1 + e, 1)$

c $x - (1 + e)y = 0$

12 a $\frac{\sec^2 x}{\tan x}$ or $\frac{1}{\sin x \cos x}$

b $-\ln |\cos x| + C$

13 a $-\frac{\cos(x^3 - \pi)}{3} + C$

b $\frac{e^{x^2}}{2} + C$

Chapter 6

Exercise 6.01

1 $x < 2$

2 $x < \frac{1}{4}$

3 $(-\infty, 0)$

4 a $x < 1.5$

b $x > 1.5$

c $x = 1.5$

5 $f'(x) = -2 < 0$ for all x

6 $y' = 3x^2 > 0$ for all $x \neq 0$

7 $(0, 0)$

8 $x = -3, 2$

9 a $(1, -4)$

b $(0, 9)$

c $(1, 1)$ and $(2, 0)$

d $(0, 1), (1, 0)$ and $(-1, 0)$

10 $(2, 0)$

11 $-1 < x < 1$

12 $(-\infty, -5) \cup (-3, \infty)$

13 a $x = 2, 5$

b $2 < x < 5$

c $x < 2, x > 5$

14 $p = -12$

15 $a = 1\frac{1}{2}, b = -6$

16 a $\frac{dy}{dx} = 3x^2 - 6x + 27$

b The quadratic function has $a > 0$

$b^2 - 4ac = -288 < 0$

So $3x^2 - 6x + 27 > 0$ for all x .

The function is monotonic increasing for all x .

17

