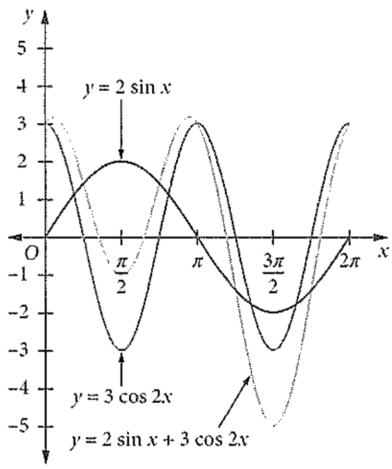
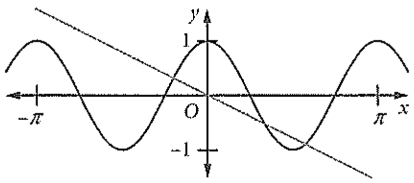
(b)



4

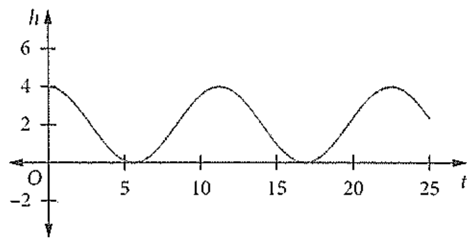


$x = -0.63, 1.07, 1.8$

5 $y = -3\cos(2x)$

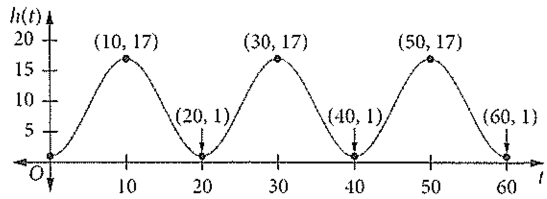
6 (a) $c = 4$
(b) $k = -8$
(c) The period of f is 24.
(d) $a = \frac{\pi}{12}$
(e) The minimum temperature is −4°C.
(f) The pond has a temperature of 0°C at 2 am and 10 am.

7 (a)



(b) The island first appears above the water at 3:16 pm.
(c) Tess can collect shells for 4 hours 43 minutes.

8 (a) Oscar is 1 m from the ground at the beginning of the ride.
(b) The greatest distance Oscar is from the ground during the ride is 17 m.
(c) The ride takes 20 s to complete one full rotation.
(d) 9 rotations are completed before the ride comes to a stop.
(e)



(f) Oscar can see the ocean for 90 seconds during the 3-minute ride.

9 (a) $x = \frac{5\pi}{8}, \frac{13\pi}{8}$ (b) $x = -\frac{3\pi}{4}, -\frac{\pi}{12}, \frac{\pi}{4}, \frac{11\pi}{12}$

CHAPTER 13

EXERCISE 13.1

1 (a) 2 (b) $\frac{1}{3}$ (c) 2.5 (d) 1 (e) 0.5 (f) $\frac{1}{9}$ (g) 3 (h) 1.5

2 (a) incorrect (b) correct (c) incorrect (d) correct

3 (a) $4\lim\limits_{x\to 0}\frac{1-\cos 2x}{(2x)^2} = \frac{4}{2} = 2$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$

(d) $\lim\limits_{x\to 0}\left(\frac{\sin x}{x} \times \sin x\right) = 1 \times 0 = 0$ (e) $\lim\limits_{x\to 0}\frac{-\sin x}{x} = -1$

(f) $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$ (g) $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$ (h) 1

EXERCISE 13.2

1 (a) $3\cos 3x$ (b) $3\cos x$ (c) $-2\sin 2x$ (d) $-2\sin x$
(e) $\cos x - 4\sin x$ (f) $2\sec^2 2x$
(g) $2\cos 2x + 2\sin 2x$ (h) $\cos\left(x + \frac{\pi}{4}\right)$

2 A

3 (a) $\cos^2 x - \sin^2 x$ (b) $\sin x + x\cos x$
(c) $2\tan x + 2x\sec^2 x$ (d) $2x\cos x - x^2\sin x$
(e) $\frac{\sin x - x\cos x}{\sin^2 x}$ (f) $\frac{x\cos x - \sin x}{x^2}$
(g) $3x^2\sin 2x + 2x^3\cos 2x$ (h) $3x^2\cos(x^3)$
(i) $\sec x + x\sec x\tan x$
(j) $\frac{x(-\operatorname{cosec} x\cot x) - \operatorname{cosec} x}{x^2} = \frac{-\operatorname{cosec} x(x\cot x + 1)}{x^2}$
(k) $2x\cot x - x^2\operatorname{cosec}^2 x$
(l) $\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{d}{dx}(\tan x) = \sec^2 x$

4 (a) $\frac{1}{2}\cos\frac{t}{2} - \frac{1}{2}\sin t$ (b) $-3\cos^2 t\sin t$ (c) $-2t\sin(t^2+1)$
(d) $2\cos\left(2t + \frac{\pi}{2}\right) = -2\sin 2t$ (e) $2t + \frac{1}{2}\sec^2\frac{t}{2}$
(f) $2t\cos 3t - 3(t^2-1)\sin 3t$ (g) $-2\sin\left(2t + \frac{\pi}{3}\right)$ (h) $-3\sin(3t-2)$

5 (a) $-4\sin 2x\cos 2x$ (b) $6\sin 3x\cos 3x$ (c) $-3\sin x\cos^2 x$
(d) $-3x^2\sin(x^3)$ (e) $2\cos^2 2x - 2\sin^2 2x$ (f) $3 - 2\sin x - \frac{1}{2}\cos\frac{x}{2}$
(g) $-4\sin x\cos x$ (h) $\frac{x^2\cos x - 2x\sin x}{x^4} = \frac{x\cos x - 2\sin x}{x^3}$
(i) $2x\sec^2(x^2-1)$ (j) $\cos\frac{x}{2} - \frac{x}{2}\sin\frac{x}{2}$
(k) $\cos x(1+\cos x) + \sin x(-\sin x) = \cos x + \cos^2 x - \sin^2 x$
(l) $-\sin 2x + \sin x + x\cos x$
(m) $-2x\operatorname{cosec}(x^2-1)\cot(x^2-1)$
(n) $2\cot(2x) \times (-\operatorname{cosec}^2(2x)) \times 2 = -4\cot(2x)\operatorname{cosec}^2(2x)$
(o) $\sec x\tan x - \operatorname{cosec} x\cot x$
(p) $\frac{d}{dx}\left(1 + \tan^2 x\right) = 2\tan x\sec^2 x$

6 (a) correct (b) incorrect (c) incorrect (d) correct

7 (a) $\frac{\cos 2x}{\sqrt{\sin 2x}}$ (b) $2(\sin^2 x - \cos^2 x)$ (c) 0 (d) $4\sin x\cos x$
(e) $-\sin x\cos(\cos x)$ (f) $-\cos x\sin(\sin x)$ (g) $-\sec^2 x$
(h) $\frac{\sin x}{2\sqrt{1-\cos x}}$ (i) $\frac{1}{2\sqrt{x}}\sec^2\sqrt{x}$

8 (a) $e^x(\sin x + \cos x)$ (b) $e^{2x}\left(2\cos\frac{x}{2} - \frac{1}{2}\sin\frac{x}{2}\right)$
(c) $e^{-x}(3\cos 3x - \sin 3x)$ (d) $e^x(\cos 4x - 4\sin 4x)$
(e) $-2e^{-x}(\sin x)$ (f) $2e^{\sin 2x}\cos 2x$ (g) $-e^{\cos x}\sin x$
(h) $(\cos x - \sin x)e^{\sin x + \cos x}$

EXERCISE 13.3

1 (a) $\frac{1}{x}$ (b) $\frac{2}{x}$ (c) $\frac{2}{x}$ (d) $\frac{3}{3x-5}$ (e) $\frac{1}{x}$ (f) $2x - \frac{4}{4x-1}$

2 (a) $x > -\frac{2}{3}, \frac{3}{3x+2}$ (b) all real x, $\frac{2x}{x^2+1}$ (c) $x \neq 2, \frac{2}{x-2}$

(d) $x > -\frac{3}{4}$, $\frac{4}{4x+3}$ (e) $x > 0$, $\frac{1}{2x}$ (f) $x > 0$, $\frac{2\sqrt{x}+1}{2x\left(\sqrt{x}+1\right)}$

3 C

4 (a) $\ln x + 1$ (b) $x^2(3\ln x + 1)$ (c) $\ln(x+2) + 1$

(d) $2x\ln 2x + x + \frac{1}{x}$ (e) $2\ln x + 2 - \frac{5}{x}$ (f) $\frac{e^x}{x}(x\ln x + 1)$

(g) $\frac{e^{2x}}{x}(2x\ln 2x + 1)$ (h) $\frac{\log_e x - 1}{(\log_e x)^2}$ (i) $\frac{1-\log_e x}{x^2}$

(j) $\frac{1 - x\log_e x}{xe^x}$

(k) $\dfrac{x \times \frac{2x}{x^2+1} - \log_e\left(x^2+1\right)}{x^2} = \dfrac{2x^2 - \left(x^2+1\right)\log_e\left(x^2+1\right)}{x^2\left(x^2+1\right)}$

(l) $e^x\log_e\left(e^x+1\right) + \frac{e^{2x}}{e^x+1}$

5 (a) $\frac{2x-2}{x^2-2x}$ (b) 1 (c) $\frac{3}{x}$ (d) $\frac{6}{x^2-9}$ (e) $\frac{2x}{x^2-1}$

(f) $\frac{4}{x}$ (g) 1 (h) $2x(2\log_e x + 1)$ (i) $\frac{3}{2x}$ (j) $\frac{1}{x\log_e x}$

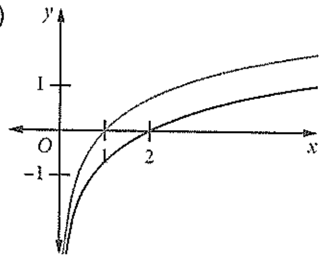
(k) $(\log_e x + 1)e^{x\log_e x}$ (l) $\frac{e^x}{e^x+1} - \frac{e^x}{e^x-1} = \frac{2e^x}{1-e^{2x}}$

6 $\frac{dy}{dx} = \frac{2x}{x^2+1}$, $m = 0.6$

7 (a) $\frac{1}{x}$ (b) $\frac{-1}{x^2}$ (c) 0.5 (d) -0.25

8 $y = x - 1$, $y = 1 - x$

9 (a)



$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

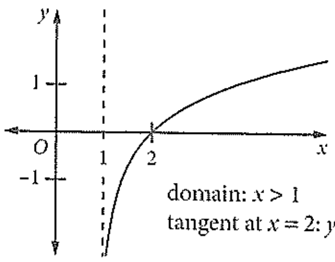
$\frac{d}{dx}\left(\log_e \frac{x}{2}\right) = \frac{1}{\frac{x}{2}} \times \frac{1}{2} = \frac{1}{x}$

The gradients are equal.

(b) $x - 2y - 2 = 0$

10 $\log_e(e^x) = x\log_e e = x$, so $y = \log_e(e^x)$ is the same as $y = x$; gradient $= 1$

11



domain: $x > 1$

tangent at $x = 2$: $y = x - 2$

12 (a) $x = \ln 2 \approx 0.693$ (b) $x = \frac{\log_e 5}{3} \approx 0.536$

(c) $x = \frac{\log_e 7 - 3}{2} \approx -0.527$ (d) $x = \pm\sqrt{1 + \ln 10} \approx \pm 1.817$

13 (a) $y = \log_e(x^3 - 1) - \log_e x$

$\frac{dy}{dx} = \frac{3x^2}{x^3-1} - \frac{1}{x} \quad \left(= \frac{2x^3+1}{x(x^3-1)}\right)$

(b) $f(x) = \log_e(e^x(x+2)) = x + \log_e(x+2)$

$f'(x) = 1 + \frac{1}{x+2}$

(c) $y = \log_e\left(\sqrt{x(x+1)^5}\right) = \frac{1}{2}\log_e x + 5\log_e(x+2)$

$\frac{dy}{dx} = \frac{1}{2x} + \frac{5}{x+1} \quad \left(= \frac{11x+1}{2x(x+1)}\right)$

(d) $f(x) = \log_e\left(\frac{e^x + x}{\sqrt{x}}\right) = \log_e(e^x + x) - \frac{1}{2}\log_e x$

$f'(x) = \frac{e^x+1}{e^x+x} - \frac{1}{2x} \quad \left(= \frac{2xe^x + x - e^x}{2x(e^x+x)}\right)$

(e) $g(x) = \log_e\left(\frac{x^3(e^x+1)}{e^{-x}+1}\right)$

$= 3\log_e x + \log_e(e^x+1) - \log_e(e^{-x}+1)$

$g'(x) = \frac{3}{x} + \frac{e^x}{e^x+1} + \frac{e^{-x}}{e^{-x}+1}$

(f) $y = \log_e \frac{\sqrt[3]{x^2 \sin x}}{1 = 2e^x} = \frac{2}{3}\log_e x + \log_e \sin x - \log_e(1 - 2e^x)$

$\frac{dy}{dx} = \frac{2}{3x} + \frac{\cos x}{\sin x} + \frac{2e^x}{1-2e^x} = \frac{2}{3x} + \cot x + \frac{2e^x}{1-2e^x}$

14 (a) $2^x\ln 2$ (b) $e^x + 3^x\ln 3$ (c) $\frac{1}{x\ln 2}$ (d) $1 + \frac{1}{x\ln 3}$

(e) $4^x(2x + x^2\ln 4)$ (f) $x^2\left(3\log_5 x + \frac{1}{\ln 5}\right)$

(g) $\frac{2^x(x\ln 2 - 1)}{x^2}$ (h) $\frac{1 - 2x\ln a\log_a x}{x^3}$

15 (a) $-a^{-x}\ln a$ (b) $= a^x\left(\ln a\log_a x + \frac{1}{x\ln a}\right)$

(c) $\frac{1 - x\log_a x \times (\ln a)^2}{xa^x\ln a}$ (d) $\frac{1}{2x\ln a\sqrt{\log_a x}}$

(e) $\frac{\sqrt{a^x}(1 + x\ln a)}{2\sqrt{x}}$ (f) $2x\log_2 x\left(\log_2 x + x \times \frac{1}{x\ln 2}\right)$

$= \frac{(2x\log_2 x)}{(\ln 2)((\ln 2)\log_2 x + 1)}$

16 (a) $(10\ln 10)x - y + 10(1 - \ln 10) = 0$

(b) $x + 25y\ln 5 - 625\ln 5 - 2 = 0$ (c) Tangents parallel at $(0, 1)$

EXERCISE 13.4

1 (a) $2xe^{x^2}$ (b) $4\left(e^x + 2x\right)\left(e^x + x^2\right)^3$ (c) $e^x + e$

(d) $-4\sin x\, e^{\cos x}$ (e) $\frac{1}{2\sqrt{x}}e^{\sqrt{x}+1}$

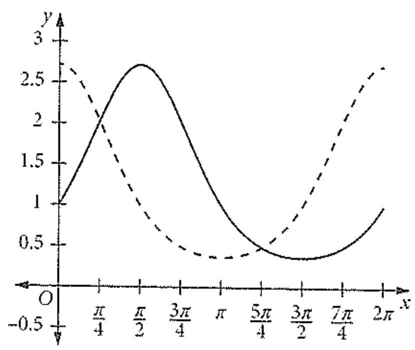
(f) $\left(1 + \frac{1}{x}\right)e^{x+\ln x} = \left(\frac{x^2+1}{x}\right)e^{x+\ln x}$

2 (a) $e^{\sin x} + x\cos x\, e^{\sin x}$ (b) $e^x\log_e x + \frac{e^x}{x}$

(c) $-2\sin(2x+1)e^{\cos(2x+1)}$ (d) $1 + 2x\,e^x + x^2e^x$

3 $\frac{dy}{dx} = \frac{750e^{-0.5t}}{\left(1 + 15e^{-0.5t}\right)^2}$

4 (a)



(b) $\left(\frac{\pi}{4}, 2.028\right), \left(\frac{5\pi}{4}, 0.493\right)$

$e^{\sin x} = e^{\cos x}$ $\sin x = \cos x$, $\tan x = 1$,

$x = \frac{\pi}{4}, \frac{5\pi}{4}$, $f\left(\frac{\pi}{4}\right) = e^{\frac{1}{\sqrt{2}}} = 2.028$, $f\left(\frac{5\pi}{4}\right) = e^{\frac{-1}{\sqrt{2}}} = 0.493$

(c) $f'(x) = \cos x\, e^{\sin x}$, $g'(x) = -\sin x\, e^{\cos x}$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} e^{\frac{1}{\sqrt{2}}}$, $g'\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} e^{\frac{1}{\sqrt{2}}}$

$f'\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} e^{\frac{-1}{\sqrt{2}}}$, $g'\left(\frac{5\pi}{4}\right) = \frac{1}{\sqrt{2}} e^{\frac{-1}{\sqrt{2}}}$

(d) No

$g'\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} e^{\frac{1}{\sqrt{2}}} = -f'\left(\frac{\pi}{4}\right)$

$g'\left(\frac{5\pi}{4}\right) = \frac{1}{\sqrt{2}} e^{\frac{-1}{\sqrt{2}}} = -f'\left(\frac{5\pi}{4}\right)$

5 0

CHAPTER REVIEW 13

1 (a) $\cos x + 2\sec^2 2x$ (b) $-12\sin 4x - 10\cos 2x$

(c) $\cos x - x\sin x$ (d) $\frac{-1}{\sin^2 x} = -\operatorname{cosec}^2 x$

(e) $-2e^{-x}(\cos 3x + 3\sin 3x)$ (f) $\frac{2\cos 2x}{\sin 2x} = 2\cot 2x$

2 (a) $(x^2 + 4x + 2)e^x$ (b) $\frac{2e^{-x}(1 - x\ln x)}{x}$ (c) $\frac{e^x}{1 + e^x}$

(d) $\frac{2x+2}{x^2+2x}$ (e) $e^{-3x}(3 - 7x - 3x^2)$ (f) $\frac{1}{2\sqrt{x}} e^{\sqrt{x}} + \frac{1}{2x}$

3 (a) 9, 6, 0, −3, 0, 6, 9 (b) $t = 4$, $v = \frac{-\pi\sqrt{3}}{2}$, $a = \frac{\pi^2}{12}$

4 (a) $2x\sin x + x^2\cos x$ (b) $\frac{2x\cos x + x^2\sin x}{\cos^2 x}$

(c) $\cos x\cos x + \sin x(-\sin x) = \cos^2 x - \sin^2 x$

(d) $\frac{\sec^2 x}{2\sqrt{\tan x}}$

(e) $2x \times (-\sin(x^2)) = -2x\sin(x^2)$

(f) $\frac{(2x+1)\cos x - 2\sin x}{(2x+1)^2}$

5 (a) $\frac{2x+2}{x^2+2x+1} = \frac{2(x+1)}{(x+1)^2} = \frac{2}{x+1}$

(b) $e^x \log_e x + \frac{e^x}{x}$ (c) $\frac{\frac{1}{x} \times e^x - \log_e x \times e^x}{e^{2x}} = \frac{1 - x\log_e x}{x e^x}$

(d) $\frac{1}{\tan x} \times \sec^2 x = \frac{1}{\sin x\cos x}$

(e) $\frac{\left(2x + \frac{1}{x}\right) \times x - \left(x^2 + \log_e x\right) \times 1}{x^2} = \frac{x^2 + 1 - \log_e x}{x^2}$

(f) $4x^3 - 2x\sin(x^2) + \cot x$

6 (a) $f(x) = \log_e x + \log_e(\tan x)$

$f'(x) = \frac{1}{x} + \frac{\sec^2 x}{\tan x} = \frac{1}{x} + \frac{1}{\sin x\cos x}$

(b) $y = \log_e(x^3 - 6) - \log_e(e^{-x} - 1)$

$\frac{dy}{dx} = \frac{3x^2}{x^3 - 6} + \frac{e^{-x}}{e^{-x} - 1}$

(c) $f(x) = \frac{1}{2}\log_e x + \log_e(\cos x) - \log_e(1 - \sin^2 x)$

$f'(x) = \frac{1}{2x} - \frac{\sin x}{\cos x} - \frac{-2\sin x\cos x}{1 - \sin^2 x} = \frac{1}{2x} + \tan x$

OR

$f(x) = \log_e \frac{\sqrt{x}\cos x}{1 - \sin^2 x} = \log_e \frac{\sqrt{x}\cos x}{\cos^2 x} = \log_e x - \log_e(\cos x)$

$f'(x) = \frac{1}{2x} - \frac{-\sin x}{\cos x} = \frac{1}{2x} + \tan x$

7 (a) $x^2\, 10^x(3 + x\ln 10)$ (b) $\cos x + \frac{1}{x\ln a}$

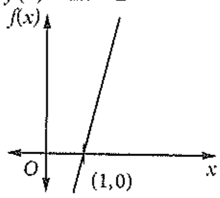
(c) $2^x \ln 2 + 3^x \ln 3 + 4^x \ln 4$ (d) $\frac{a^x\left(x(\ln a)^2 \log_a x - 1\right)}{x\ln a\left(\log_a x\right)^2}$

CHAPTER 14

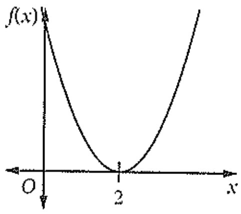
EXERCISE 14.1

1 A, C

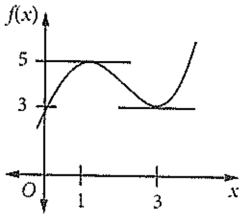
2 $f(x) = 2x - 2$



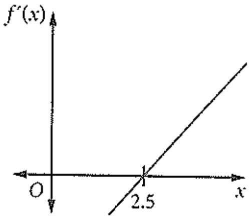
3



4



5

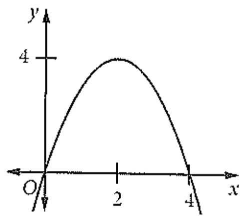


$f'(x) = 2x - 5$

(a) $x < 2.5$ (b) $x = 2.5$

(c) $x > 2.5$

6



$x = 2$; positive, negative

7 (a) $x < -1.5$ (b) $x > -1.5$ (c) $x = -1.5$

8 (a) $f'(x) = 3x^2 - 12x + 9$ (b) $x < 1, x > 3$

(c) $1 < x < 3$ (d) $x = 1$

9 (a) real x (b) none (c) never

10 (a) $x = -\frac{1}{3}, 1$ (b) $x < -\frac{1}{3}, x > 1$ (c) $-\frac{1}{3} < x < 1$

EXERCISE 14.2

1 (a) $f'(x) = 2x - 6$

(b) $(3, -1)$ minimum turning point

(c)