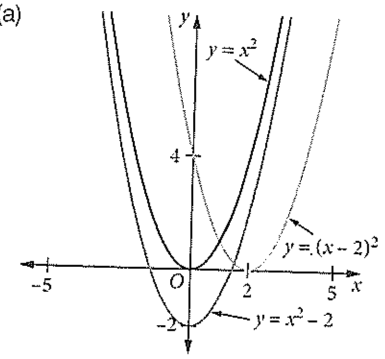
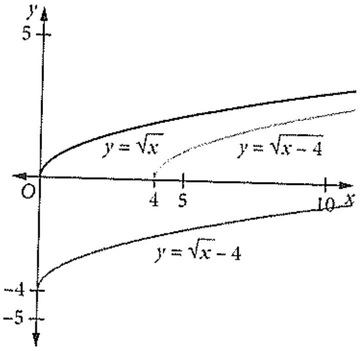
CHAPTER 15

EXERCISE 15.1

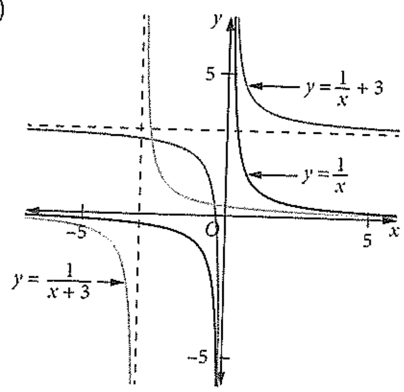
1 (a)



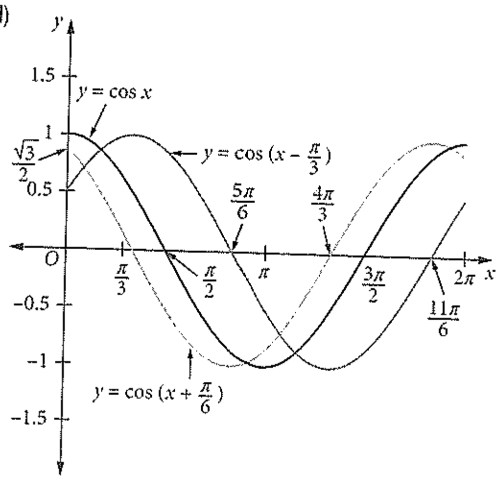
(b)



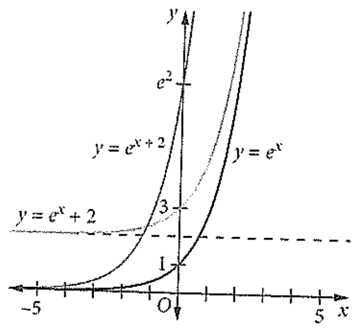
(c)



(d)



2 (a)



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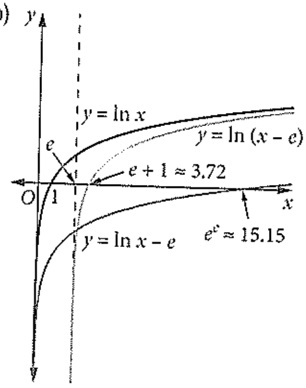

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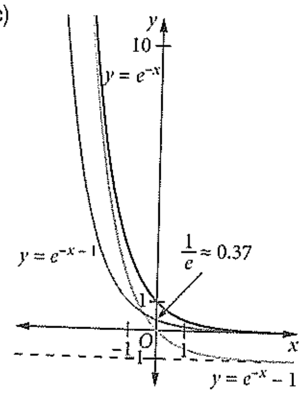
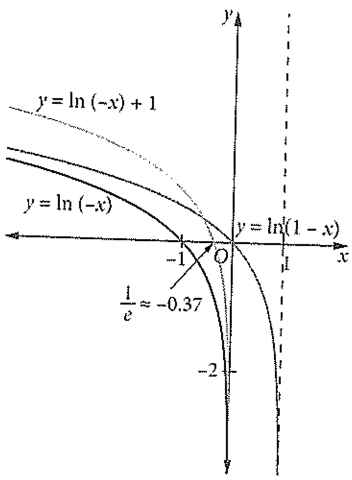
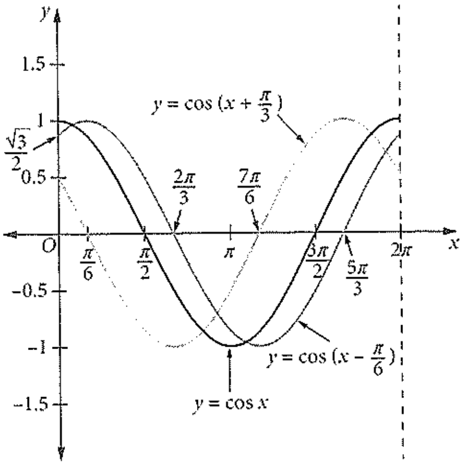


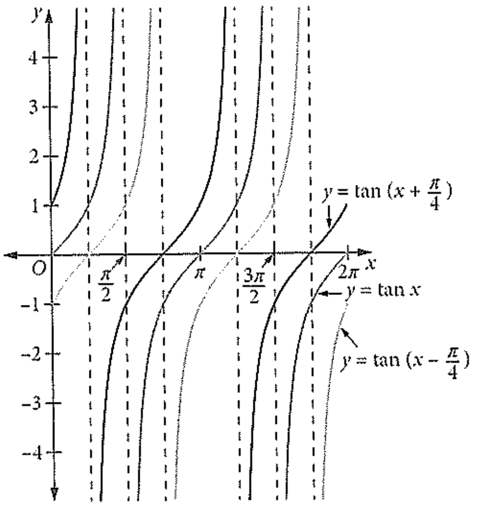
(d)



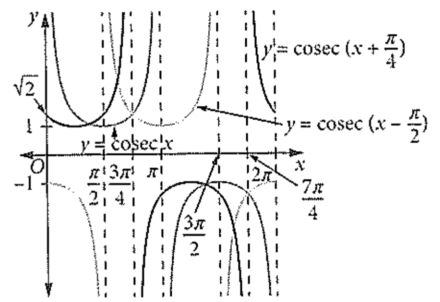
3 (a)



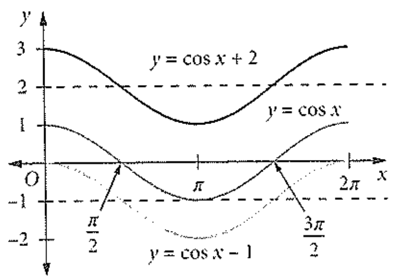
(b)



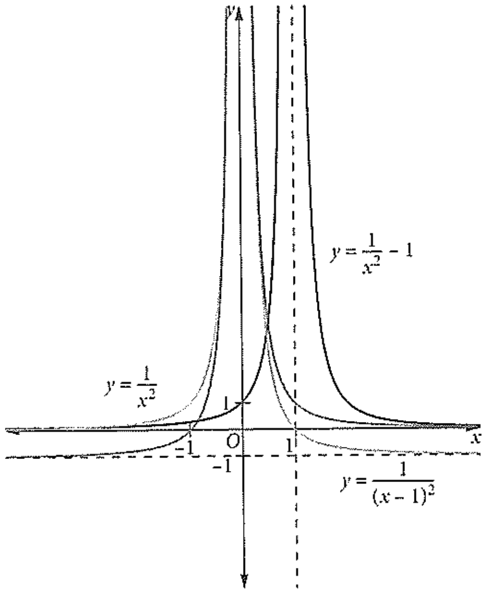
(c)



(d)



4 (a)



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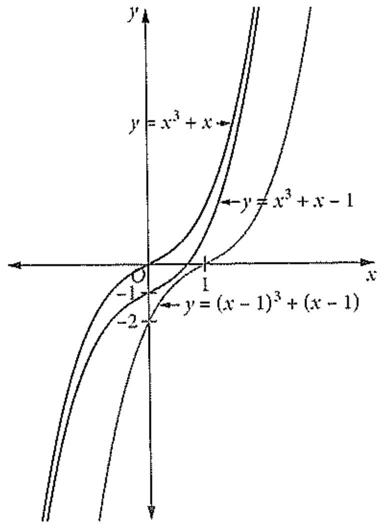
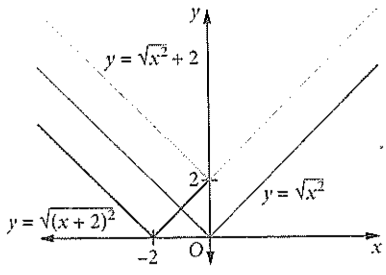


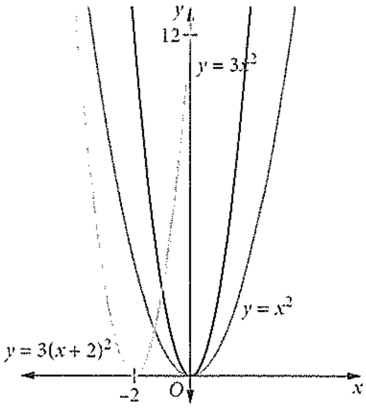
(c)



5 C.

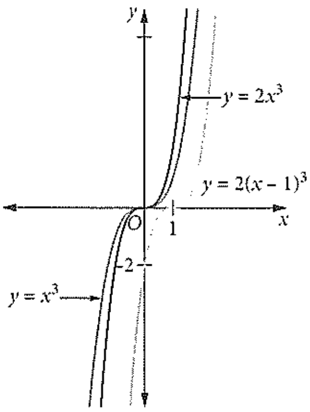
EXERCISE 15.2

1 (a)



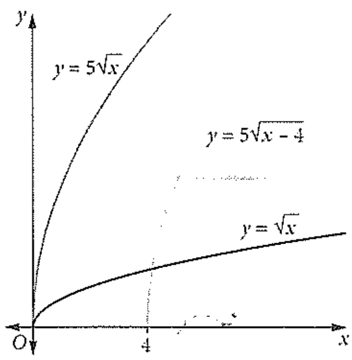
The dilation from the x-axis for the second and third graphs has factor 3.

(b)



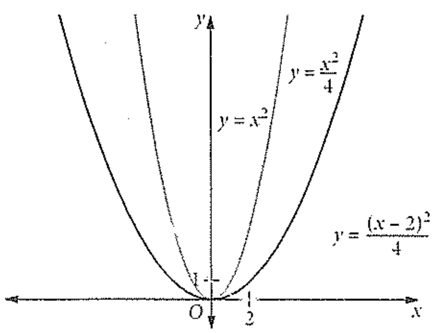
The dilation from the x-axis for the second and third graphs has factor 2.

(c)



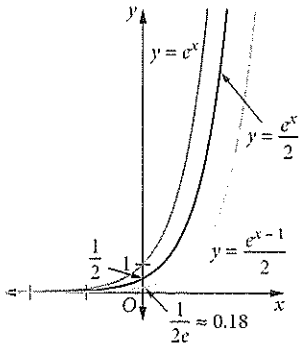
The dilation from the x-axis for the second and third graphs has factor 5.

(d)



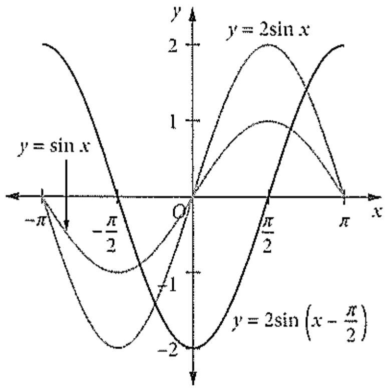
The dilation from the x-axis for the second and third graphs has factor $\frac{1}{4}$.

2 (a)



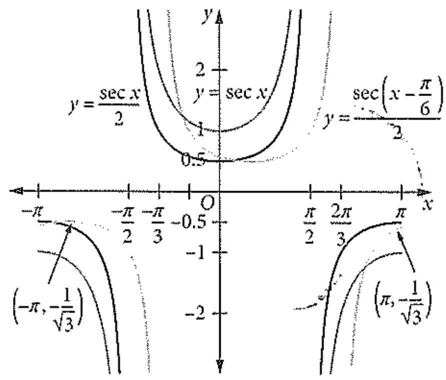
The dilation from the x-axis for the second and third graphs has factor $\frac{1}{2}$.

(b)



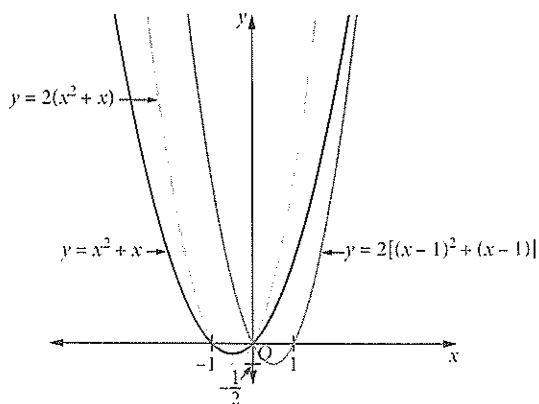
The dilation from the x-axis for the second and third graphs has factor 2.

(c)



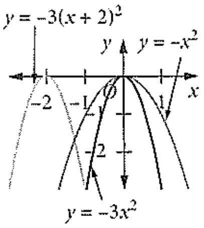
The dilation from the x-axis for the second and third graphs has factor $\frac{1}{2}$.

3 (a)



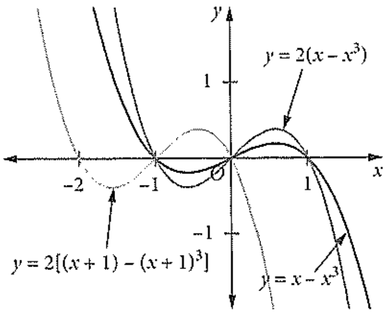
The dilation from the x-axis for the second and third graphs has factor 2.

(b)



The dilation from the x-axis for the second and third graphs has factor 3.

(c)

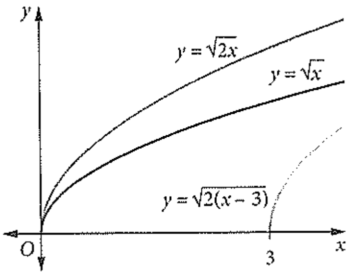


The dilation from the x-axis for the second and third graphs has factor 2.

4 B

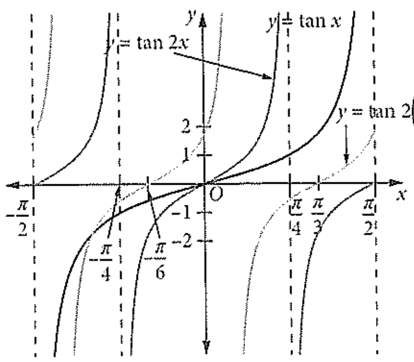
EXERCISE 15.3

1 (a)



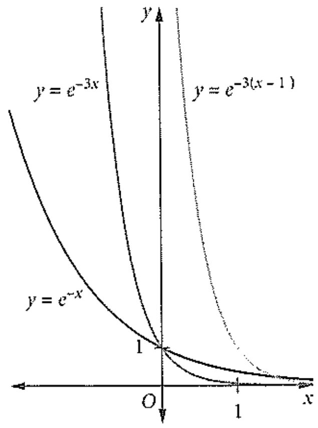
The dilation from the y-axis in the second and third graphs has a factor of 0.5. The third graph has also undergone a translation of 3 units to the right.

(b)



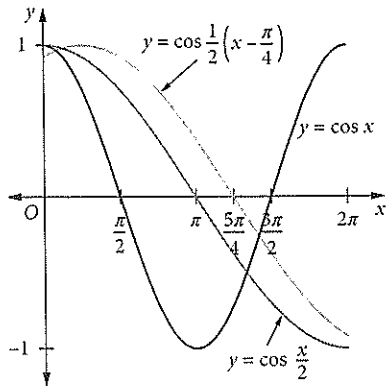
The dilation from the y-axis in the second and third graphs has a factor of 0.5. The third graph has also undergone a translation of $\frac{\pi}{6}$ units to the left.

(c)



The dilation from the y-axis in the second and third graphs has a factor of $\frac{1}{3}$. The third graph has also undergone a translation of 1 unit to the right.

(d)

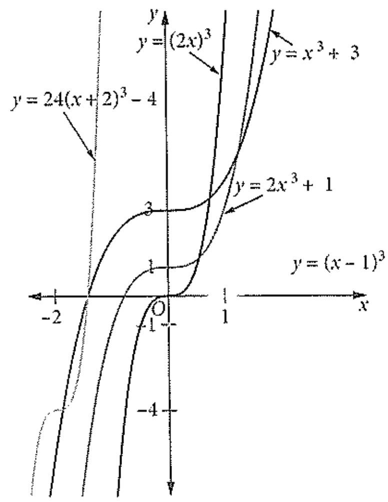


The dilation from the y-axis in the second and third graphs has a factor of 2. The third graph has also undergone a translation of $\frac{\pi}{4}$ units to the right.

2 (a) $f(2x) = (2x)^3 = 8x^3$ (b) $f(x-1) = (x-1)^3$
(c) $f(x) + 3 = x^3 + 3$ (d) $2f(x) + 1 = 2x^3 + 1$
(e) $3f(2(x+2)) - 4 = 3[2(x+2)]^3 - 4 = 24(x+2)^3 - 4$

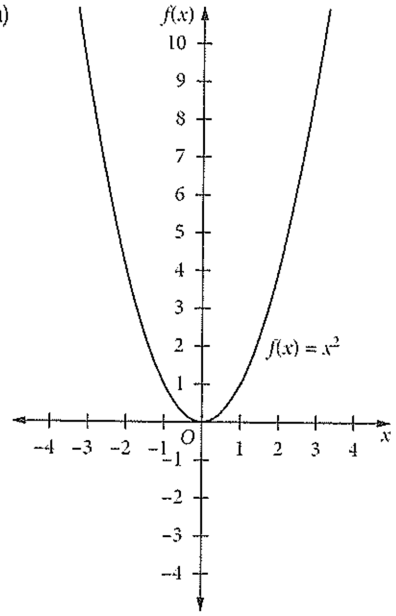
3 (a) $f(2x) = \cos x$ (b) $f\left(x + \frac{\pi}{3}\right) = \cos\frac{1}{2}\left(x + \frac{\pi}{3}\right)$
(c) $2f(x) = 2\cos\frac{x}{2}$ (d) $f(x) - 1 = \cos\frac{x}{2} - 1$
(e) $2f\left(x + \frac{\pi}{6}\right) + 1 = 2\cos\frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$

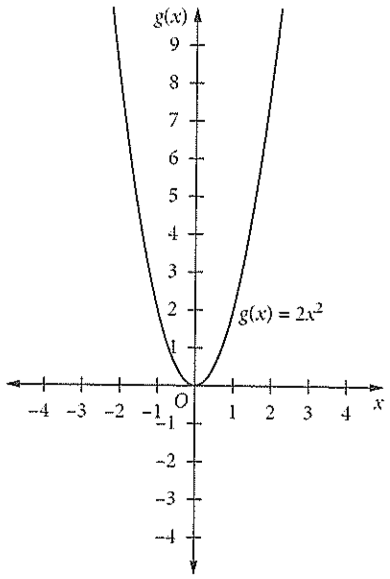
4 (a)–(e)

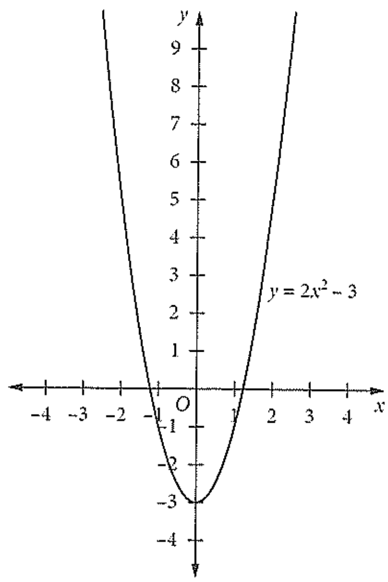


5 A

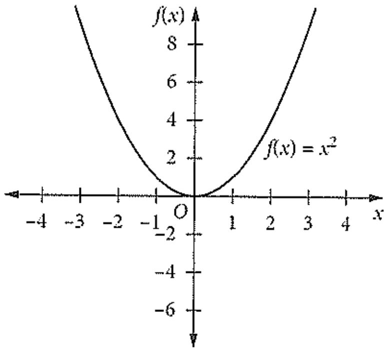
6 (a)

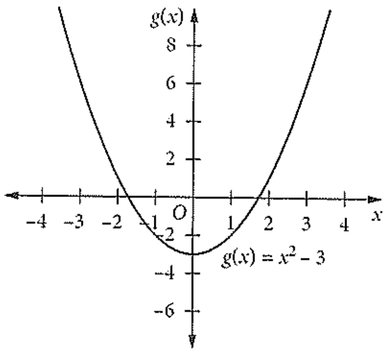


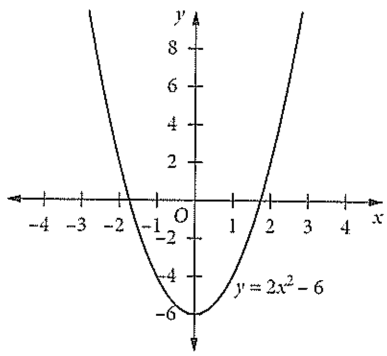




(b)

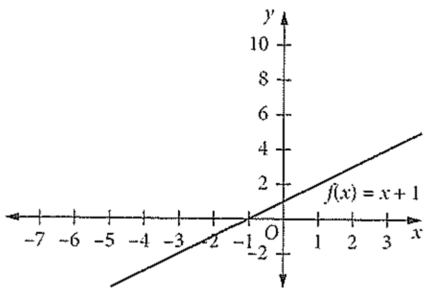


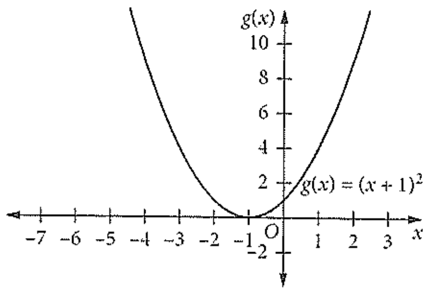


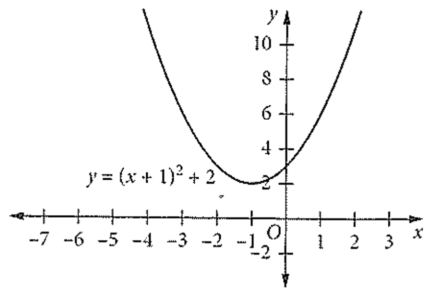


(c) The final graph in part (b) has been moved down 6 units, whereas part (a) was moved down 3 units. They have the same dilation.

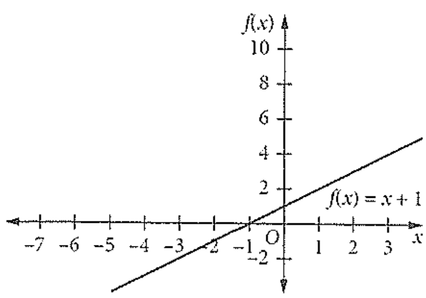
7 (a)

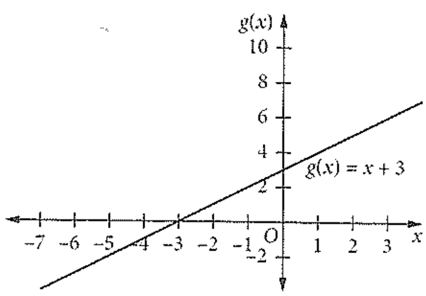


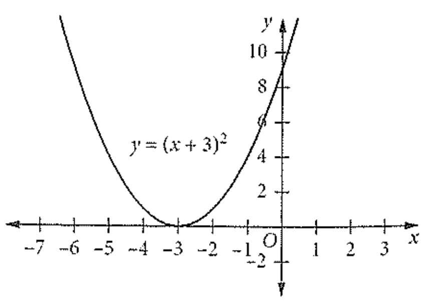




(b)

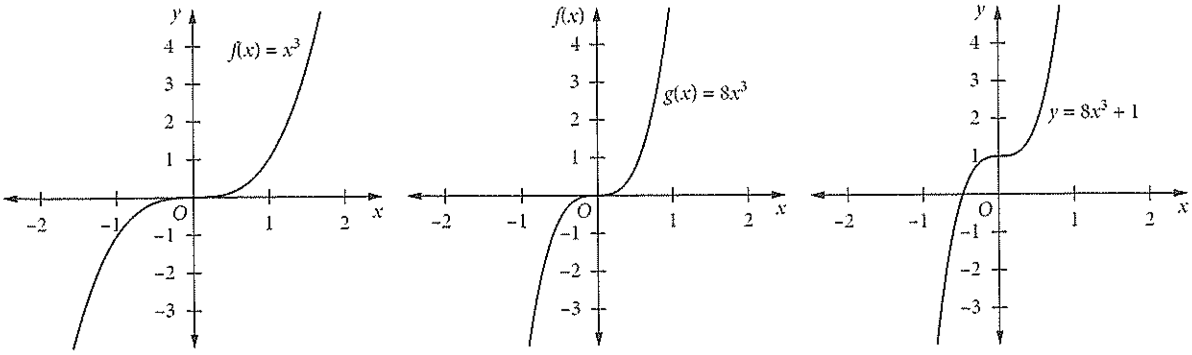




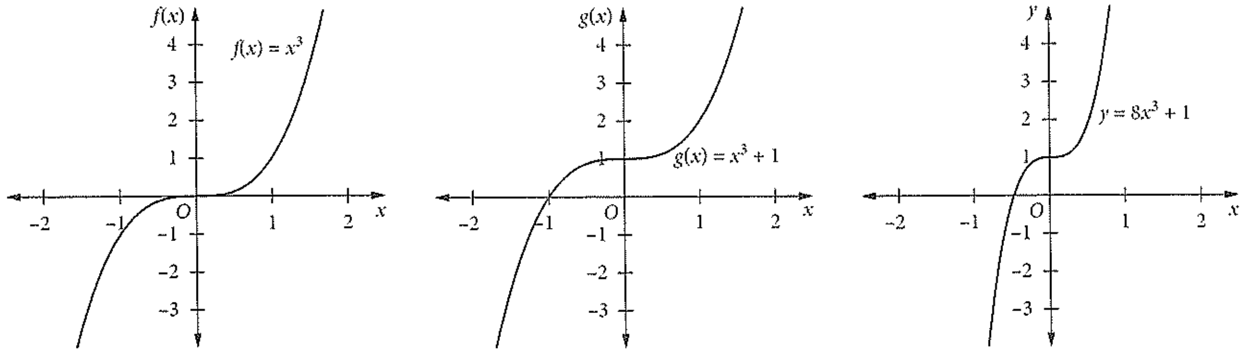


(c) Both curves are parabolas. The vertex in part (a) is at $(-1, 2)$, whereas the vertex in part (b) is at $(-3, 0)$.

8 (a)

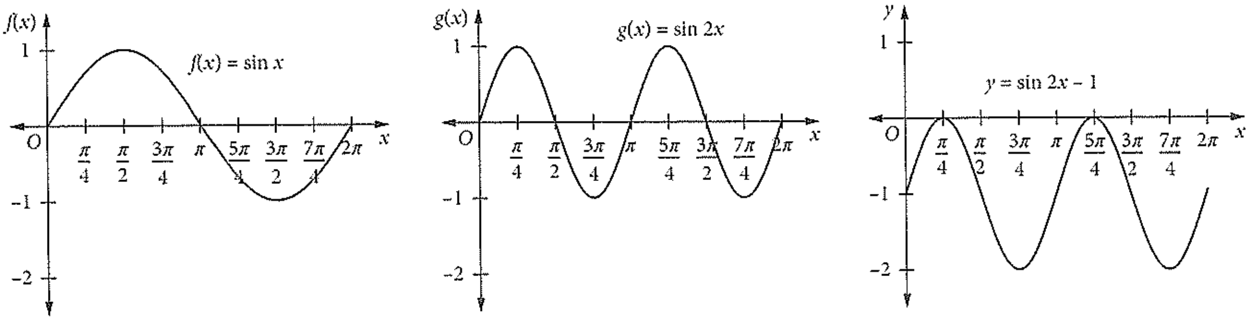


(b)

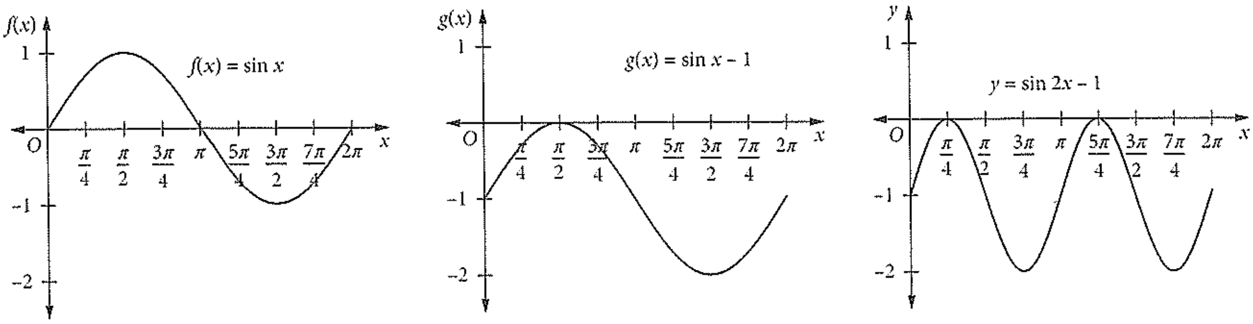


(c) The final graph is the same in each case.

9 (a)

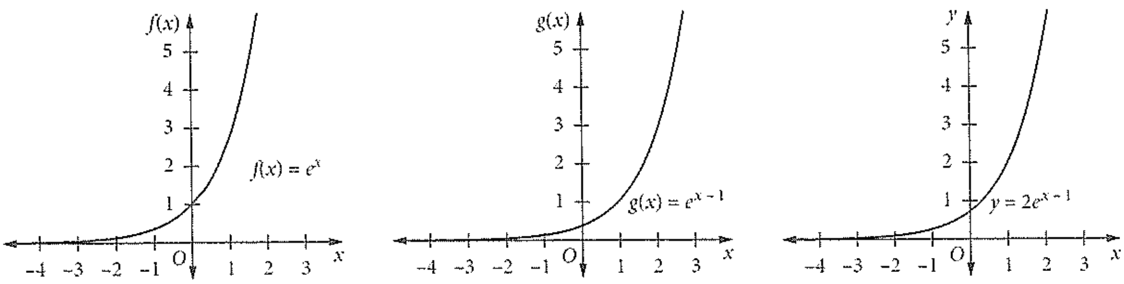


(b)

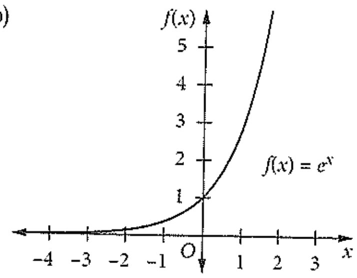


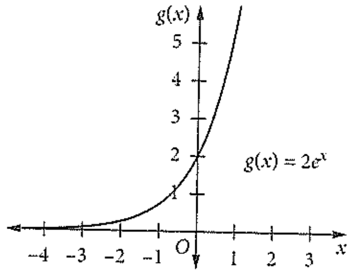
(c) The final graph is the same in each case.

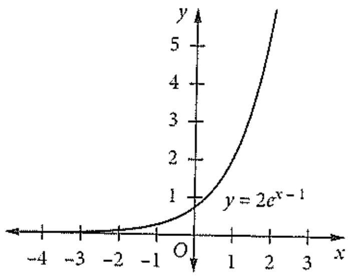
10 (a)



(b)





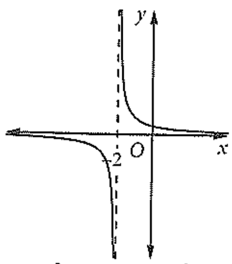


(c) The final graph is the same in each case.

EXERCISE 15.4

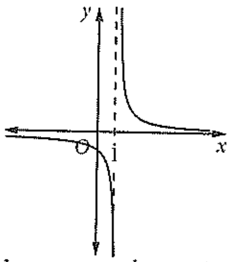
1 A

2 (a)



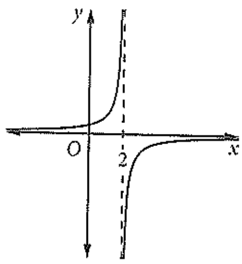
$x < -2$ range: real y, $y \neq 0$

(b)



$x < 1$ range: real y, $y \neq 0$

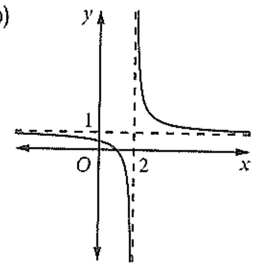
(c)



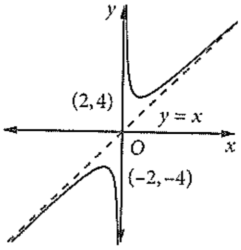
$x > 2$ range: real y, $y \neq 0$

3 (a) $1 + \frac{1}{x-2} = \frac{x-2+1}{x-2} = \frac{x-1}{x-2}$

(b)

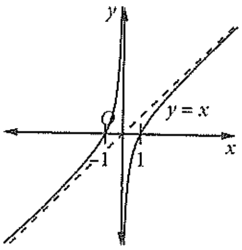


4 (a)



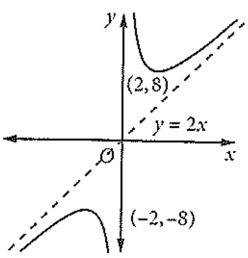
no inflections, $x > 0$
range: real y, $|y| \geq 2$

(b)



no turning points,
no inflections, $x < 0$
range: real y

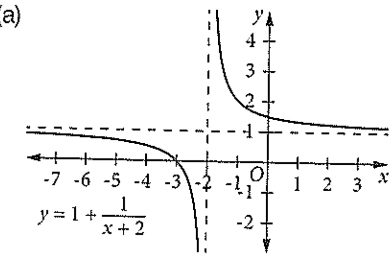
(c)



no inflections, $x > 0$
range: real y, $|y| \geq 8$

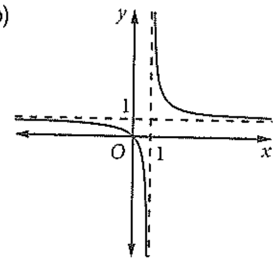
5 (a) correct (b) incorrect (c) correct (d) correct

6 (a)



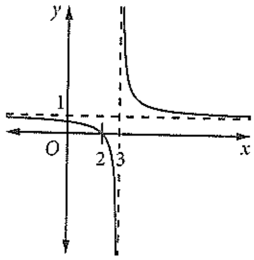
$x < -2$ range: real y

(b)



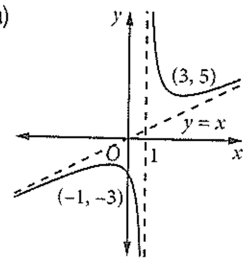
$x < 1$ range: real y, $y \neq 1$

(c)



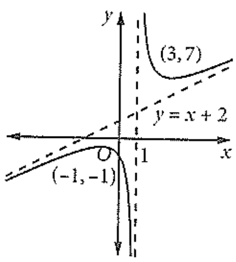
$x < 3$ range: real y, $y \neq 1$

7 (a)



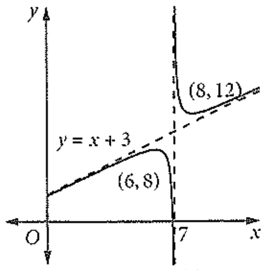
range: $y \leq -3$, $y \geq 5$

(b)



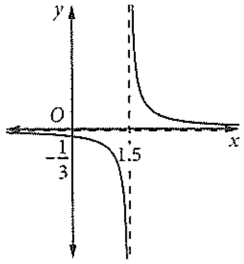
range: $y \leq -1$, $y \geq 7$

(c)



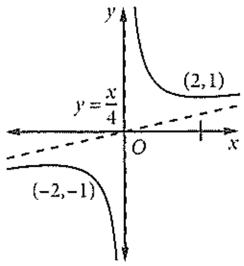
range: $y \le -8$, $y \ge 12$

(d)



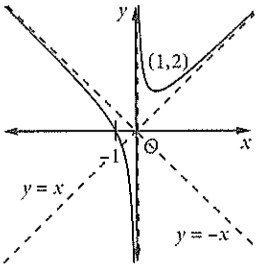
range: real y, $y \neq 0$

(e)



range: $y \le -1$, $y \ge 1$

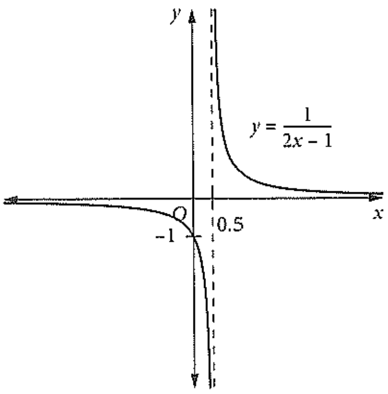
(f)



range: real y

EXERCISE 15.5

1 (a)



(b) $\frac{dy}{dx} = \frac{-2}{(2x-1)^2}$

$x = 1$: $\frac{dy}{dx} = -2$

$x = 1, y = 1$

Equation of tangent: $y - 1 = -2(x - 1)$

$2x + y - 3 = 0$

(c) $x = -1$: $\frac{dy}{dx} = -\frac{2}{9}$

Gradient of normal $= \frac{9}{2}$

$x = -1, y = -\frac{1}{3}$

Equation of normal: $y + \frac{1}{3} = \frac{9}{2}(x + 1)$

$6y + 2 = 27x + 27$

$27x - 6y + 25 = 0$

(d) $2x + y - 3 = 0$ [1]

$27x - 6y + 25 = 0$ [2]

[1] × 6: $12x + 6y - 18 = 0$ [3]

[2] + [3]: $39x + 7 = 0$

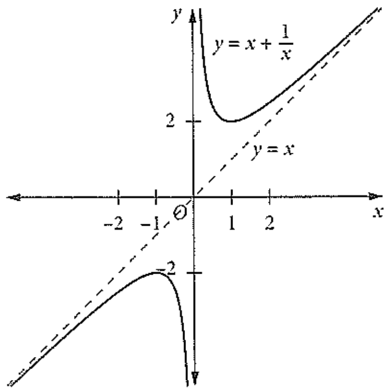
$x = -\frac{7}{39}$

Substitute into [1]: $-\frac{14}{39} + y - 3 = 0$

$y = \frac{131}{39}$

Point of intersection is $\left(-\frac{7}{39}, \frac{131}{39}\right)$

2 (a)



(b) $\frac{dy}{dx} = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2} = \frac{(x+1)(x-1)}{x^2}$

For stationary points, $\frac{dy}{dx} = 0$: $x = \pm 1$

$x = 1, y = 2$. $x = -1, y = -2$.

$\frac{d^2y}{dx^2} = \frac{2}{x^3}$

$x = 1$: $\frac{d^2y}{dx^2} = 2 > 0$

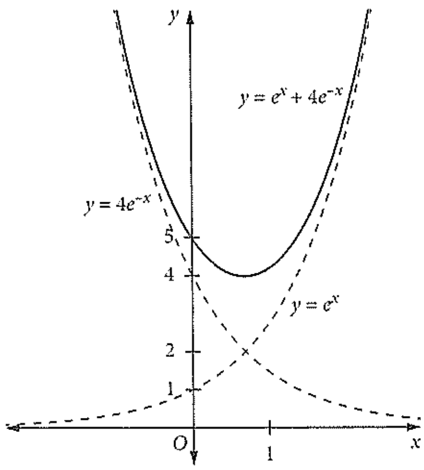
Minimum turning point at (1, 2)

$x = -1$: $\frac{d^2y}{dx^2} = -2 < 0$

Maximum turning point at (–1,–2)

(c) 2

3 (a)



On the left the curve $y = 4e^{-x}$ is the asymptote, on the right the curve $y = e^x$ is the asymptote.

(b) $f'(x) = e^x - 4e^{-x} = \frac{e^{2x} - 4}{e^x}$

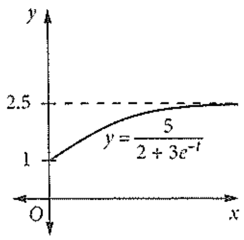
For stationary points, $\frac{dy}{dx} = 0$: $e^{2x} = 4$, $e^x = \pm 2$. Since $e^x > 0$, $e^x = 2$ is the only solution.

$e^x = 2 \rightarrow x = \ln 2$

$f'(x) > 0$ for $x > \ln 2$

(c) The minimum value of $f(x)$ is 4 and it occurs when $x = \ln 2$.

4 (a)



(b) $f'(t) = \dfrac{15e^{-t}}{\left(2+3e^{-t}\right)^2}$

$e^{-t} > 0$ for all values of t, the denominator is always positive so $f'(t) > 0$ for $t \geq 0$.

(c) As $t \to \infty$ then $f(t) \to 2.5$.

(d) $1 \leq f(t) < 2.5$

5 (a) $f(0) = 1, f\left(\frac{\pi}{2}\right) = 0, f(\pi) = -e^{-\pi}$

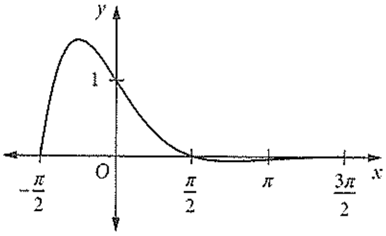
(b) $f'(x) = -e^{-x}\cos x - e^{-x}\sin x = -e^{-x}(\cos x + \sin x)$

(c) $f'(0) = -1 \times 1 - 1 \times 0 = -1$

$$f'\left(\frac{3\pi}{4}\right) = -e^{-\frac{3\pi}{4}\left(\cos\frac{3\pi}{4}+\sin\frac{3\pi}{4}\right)} = -e^{-\frac{3\pi}{4}\left(\frac{-1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)} = 0$$

$$f'\left(-\frac{\pi}{4}\right) = -e^{\frac{\pi}{4}\left(\cos-\frac{\pi}{4}+\sin-\frac{\pi}{4}\right)} = -e^{-\frac{\pi}{4}\left(\frac{1}{\sqrt{2}}+\frac{-1}{\sqrt{2}}\right)} = 0$$

(d)



(e) $f''(x) = e^{-x}(\cos x + \sin x) - e^{-x}(-\sin x + \cos x) = 2e^{-x}\sin x$

$x = -\frac{\pi}{4}, f''(x) = 2e^{\frac{\pi}{4}}\sin\left(-\frac{\pi}{4}\right) = -\sqrt{2}e^{\frac{\pi}{4}} < 0$

Maximum turning point when $x = -\frac{\pi}{4}$.

$f\left(-\frac{\pi}{4}\right) = e^{\frac{\pi}{4}}\cos\left(-\frac{\pi}{4}\right) = \dfrac{e^{\frac{\pi}{4}}}{\sqrt{2}} \approx 1.55$

6 (a) $f(x) = \log_e(\sin x)$. Require $\sin x > 0$, so $0 < x < \pi$.

(b) $f'(x) = \dfrac{\cos x}{\sin x} = \cot x$. Domain is $0 < x < \pi$.

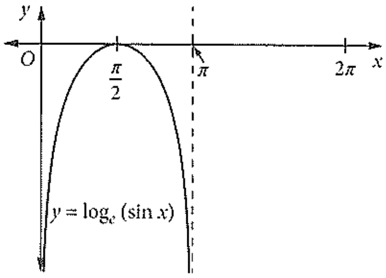
(c) $f'(x) = 0$ when $\cot x = 0$ so $x = \frac{\pi}{2}$.

$f''(x) = -\operatorname{cosec}^2 x$

$f''\left(\frac{\pi}{2}\right) = -\operatorname{cosec}^2\frac{\pi}{2} = -1 < 0$

Maximum value of $f(x)$ when $x = \frac{\pi}{2}$ is $\log_e\left(\sin\frac{\pi}{2}\right) = 0$

(d)



7 (a) $y = \log_e(1 + \sin x)$

$\dfrac{dy}{dx} = \dfrac{\cos x}{1+\sin x}$

$\dfrac{d^2y}{dx^2} = \dfrac{-\sin x(1+\sin x) - \cos x \times \cos x}{(1+\sin x)^2}$

$= \dfrac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} = \dfrac{-(1+\sin x)}{(1+\sin x)^2} = \dfrac{-1}{1+\sin x}$

(b) $\dfrac{dy}{dx} = 0$ when $\cos x = 0, x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ but y is undefined for $x = \frac{3\pi}{2}, \frac{7\pi}{2}$.

Hence $x = \frac{\pi}{2}, \frac{5\pi}{2}$

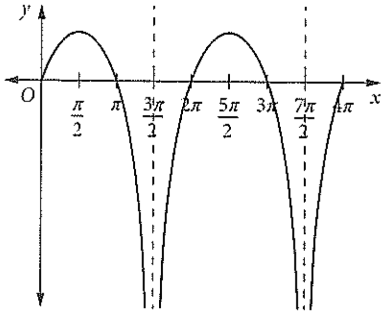
(c) $x = \frac{\pi}{2}$: $\dfrac{d^2y}{dx^2} = \dfrac{-1}{1+1} = -\frac{1}{2} < 0$ so maximum when $x = \frac{\pi}{2}$.

$x = \frac{5\pi}{2}$: $\dfrac{d^2y}{dx^2} = \dfrac{-1}{1+1} = -\frac{1}{2} < 0$ so maximum when $x = \frac{5\pi}{2}$.

(d) $\dfrac{-1}{1+\sin x} \neq 0$ for any value of x so $\dfrac{d^2y}{dx^2} \neq 0$ for any value of x.

Hence no points of inflection.

(e)



8 $C(t) = 1000\left[\cos\left(\frac{\pi(t-8)}{2}\right) + 2\right]^2 - 1000$, for $8 \leq t \leq 16$

(a) $\dfrac{dC}{dt} = 1000 \times 2\left[\cos\left(\frac{\pi(t-8)}{2}\right) + 2\right] \times \left[-\sin\left(\frac{\pi(t-8)}{2}\right)\right] \times \frac{\pi}{2}$

$= -1000\pi\sin\left(\frac{\pi(t-8)}{2}\right)\left[\cos\left(\frac{\pi(t-8)}{2}\right) + 2\right]$

$\dfrac{dC}{dt} = 0$: $\sin\left(\frac{\pi(t-8)}{2}\right) = 0$ or $\cos\left(\frac{\pi(t-8)}{2}\right) = -2$

$\dfrac{\pi(t-8)}{2} = 0, \pi, 2\pi, 3\pi, 4\pi$

$t - 8 = 0, t - 8 = 2, t - 8 = 4, t - 8 = 6, t - 8 = 8$

$t = 8, 10, 12, 14, 16$

$t = 8, 12, 16$: $\cos\left(\frac{\pi(t-8)}{2}\right) = 1$

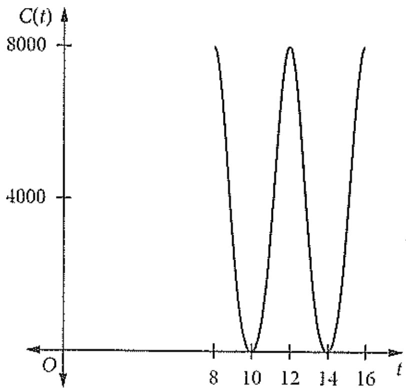
$t = 10$: $\cos\left(\frac{\pi(t-8)}{2}\right) = \cos\pi = -1$

$t = 14$: $\cos\left(\frac{\pi(t-8)}{2}\right) = \cos 3\pi = -1$

The least value of $C(t)$ will occur when $t = 10, 14$.

Minimum concentration $= 1000[\cos\pi + 2]^2 - 1000 = 0$

(b)



9 (a) $h^2 + \left(\frac{x}{2}\right)^2 = 1.69x^2$

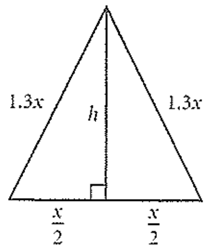
$h^2 = 1.69x^2 - 0.25x^2$

$h^2 = 1.44x^2$

$h = 1.2x$

(b) $A = xy + \frac{1}{2} \times x \times 1.2x$

$= xy + 0.6x^2$



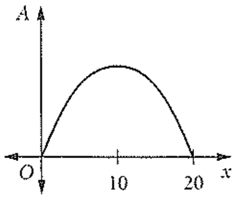
(c) $2y + x + 2.6x = 48$

$2y + 3.6x = 48$

$y = 24 - 1.8x$

(d) $A(x) = x \times (24 - 1.8x) + 0.6x^2$
$= 24x - 1.2x^2$

(e)



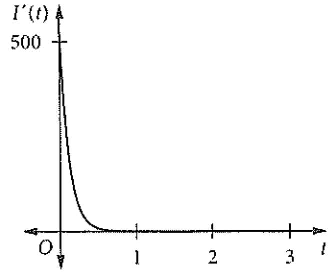
(f) Maximum area when $x = 10$ m, $y = 6$ m, equal sides of the isosceles triangle 13 m.
Maximum area = 120 m^2

10 $I(t) = 100(1 - e^{-5t})$

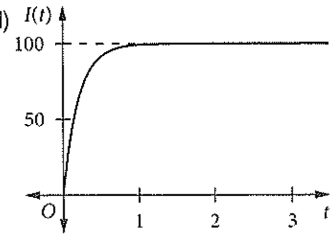
(a) $I(0) = 100(1 - 1) = 0$
$I(0.2) = 100(1 - e^{-1}) = 63.2$ amps
$I(1) = 100(1 - e^{-5}) = 99.3$ amps

(b) As t increases the current approaches 100 amps.

(c) $I'(t) = 100 \times 5e^{-5t} = 500e^{-5t}$



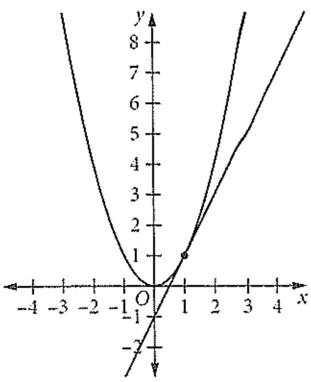
(d)



(e) The graph of $I'(t)$ shows the gradient of $I(t)$ over time. The gradient graph shows the current increasing rapidly at the start then staying practically the same.

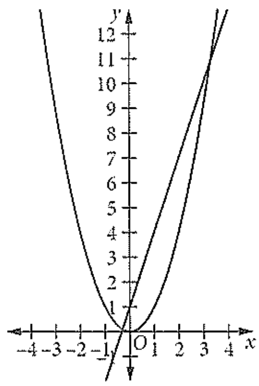
EXERCISE 15.6

1 (a)



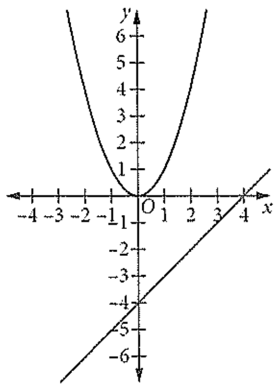
Graphs touch, equation has one solution.

(b)



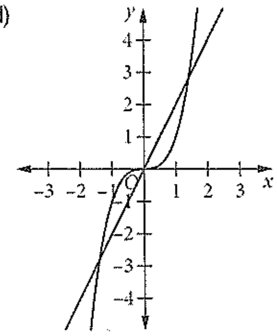
Graphs intersect twice, equation has two solutions.

(c)



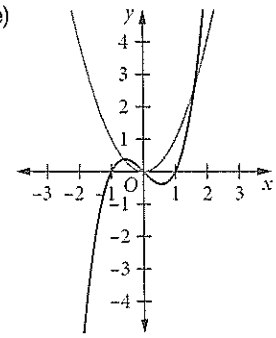
Graphs do not intersect so equation has no solutions.

(d)



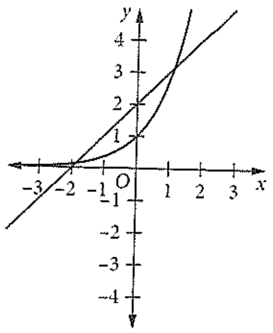
Graphs intersect 3 times. Equation has 3 solutions.

(e)



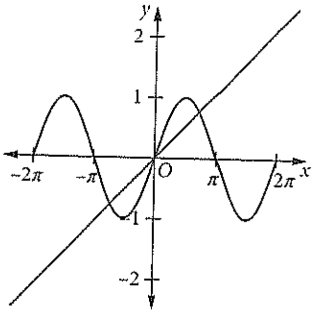
Graphs intersect 3 times. Equation has 3 solutions.

(f)



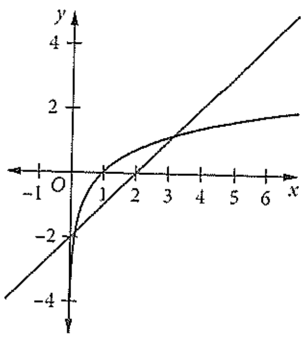
Graphs intersect twice, equation has two solutions.

2 (a)



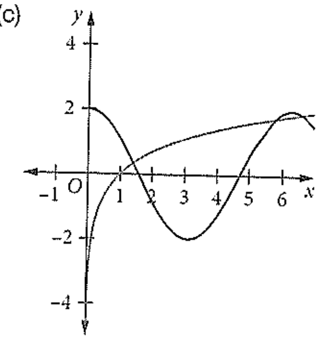
Graphs intersect 3 times. Equation has 3 solutions.

(b)



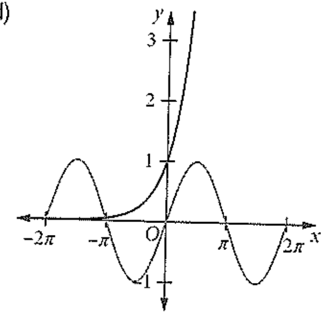
Graphs intersect twice, equation has two solutions.

(c)



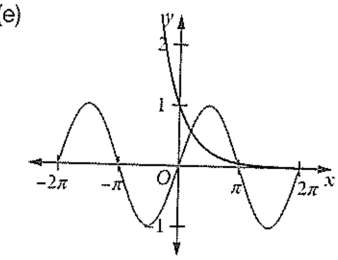
Graphs intersect twice, equation has two solutions.
If the domain of $2\cos x$ had been $0 \le x \le 3\pi$, the equation would have had 3 solutions.

(d)



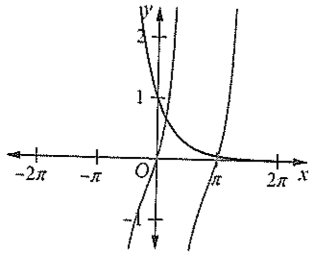
Graphs intersect 3 times, equation has 3 solutions.

(e)



Graphs intersect twice, equation has two solutions.

(f)

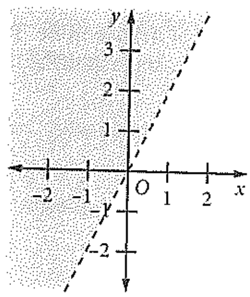


Graphs intersect twice, equation has two solutions.

3 2.84

EXERCISE 15.7

1



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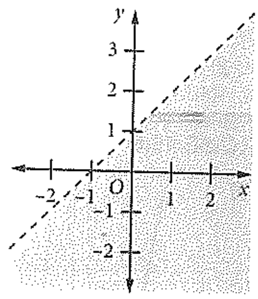

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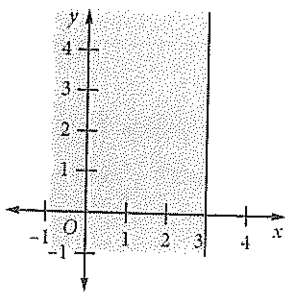

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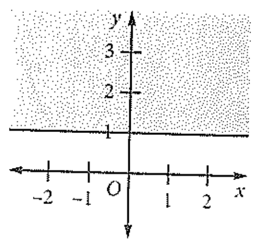

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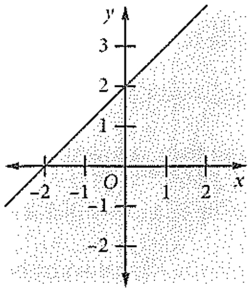

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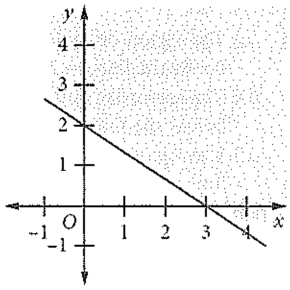

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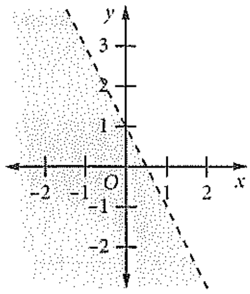

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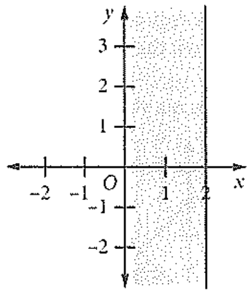

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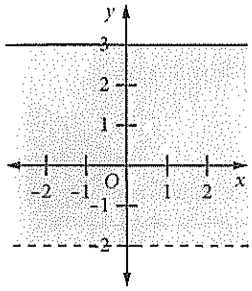

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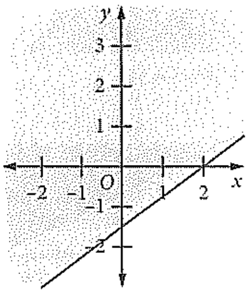

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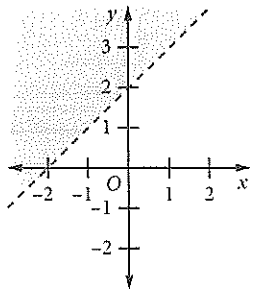

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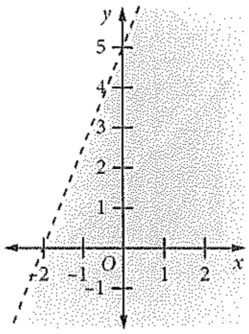

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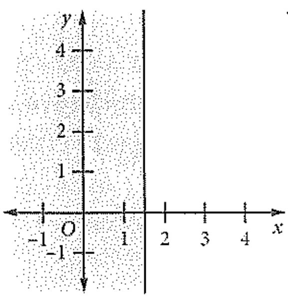

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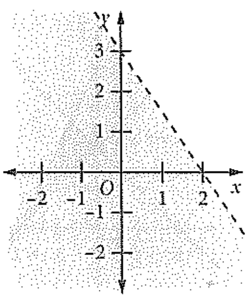

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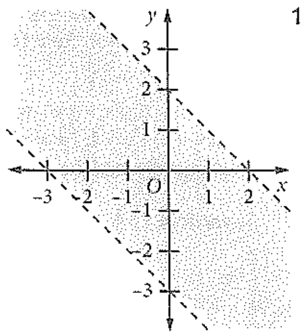

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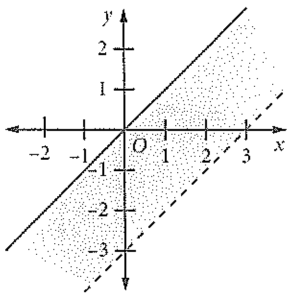
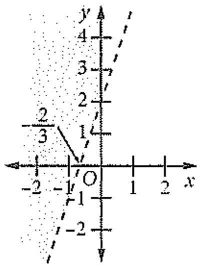


17 A

18 (a)


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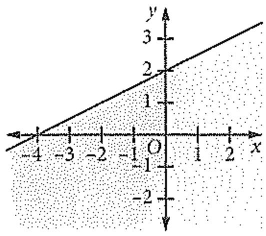
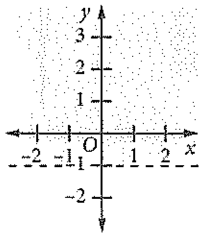


(c)


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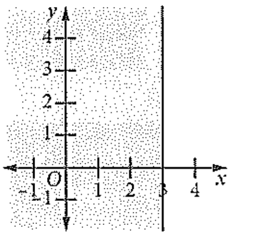

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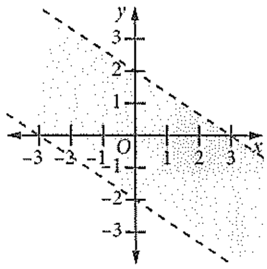
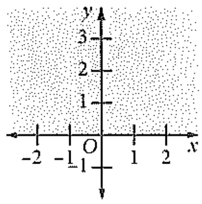
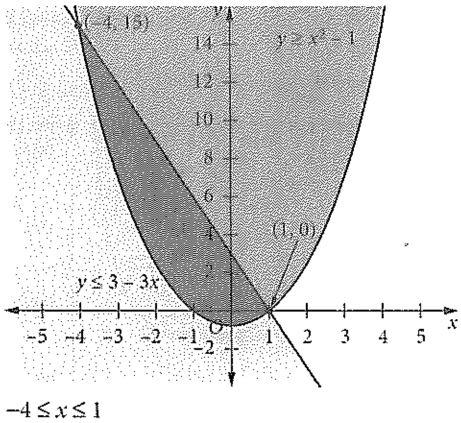


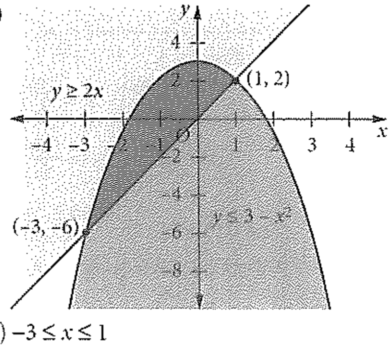
(f)


19 (a) The region below the line $y = x + 2$.
(b) The region on and above the line $y = x$.
(c) The region to the right of the line $x = 3$.
(d) The region on and below the line $y = 4$.
(e) The region on and below the line $x + 3y = 9$.
(f) The region on and to the right of the line $x = -2$ that is also to the left of the line $x = 3$.

20 (a) correct (b) incorrect (c) incorrect (d) correct

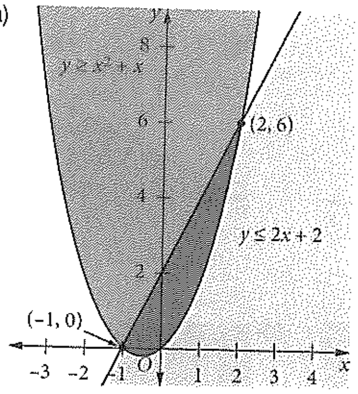
21 (a)


(b) $-4 \le x \le 1$

22 (a)


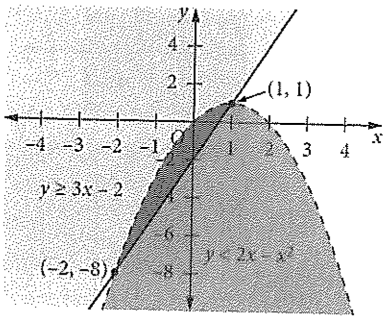
(b) $-3 \le x \le 1$

23 (a)



(b) $-1 \le x \le 2$

24 (a)



(b) $x^2 + x - 2 < 0$

$3x - 2 < 2x - x^2$

$-2 < x < 1$

(c) $x^2 + x - 2 < 0$

$(x+2)(x-1) < 0$

Roots: $x = -2, 1$

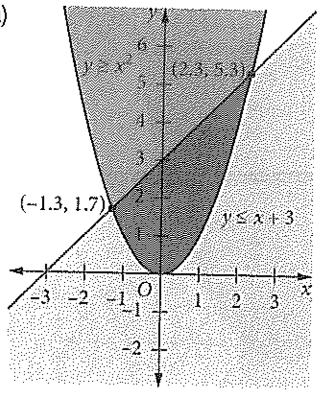
test using $x = 0$

$0 + 0 - 2 < 0$

$-2 < 0$: true

Solution is $-2 < x < 1$

25 (a)



(b) $x^2 \le x + 3$

$x^2 - x - 3 \le 0$

$-1.3 \le x \le 2.3$

(c) Solve $x^2 - x - 3 = 0$

$x = \dfrac{1 \pm \sqrt{13}}{2}$

test using $x = 0$

$0 - 0 - 3 < 0$

$-3 < 0$: true

Solution is $\dfrac{1 - \sqrt{13}}{2} \le x \le \dfrac{1 + \sqrt{13}}{2}$

$-1.303 \le x \le 2.303$

EXERCISE 15.8

1 (a) The region on and to the right of the line $x = -1$ that is also below the line $y = 2$.

(b) The region on and above the line $x + y = 1$ that is also on and below the line $y = x + 1$.

(c) The region above the line $y = 1$ that is also above the line $x + y = 1$.

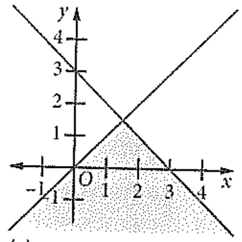
(d) The region on and to the right of the line $x = -1$ that is also to the left of the line $x = 2$.

(e) The region on and below the line $y = 2x + 2$ that is also below the line $x + y = 2$.

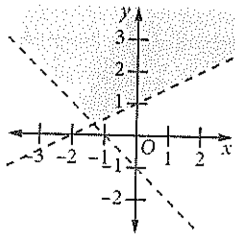
(f) The region on and above the line $y = x$ that is also on and below the line $y = 2x$.

2 C   3 $x + y \ge 3$, $y \le x + 1$

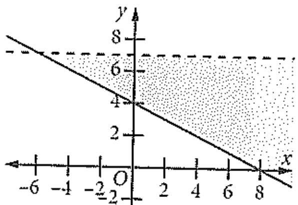
4 (a) yes, no, yes



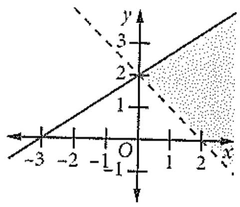
(b) no, no, yes



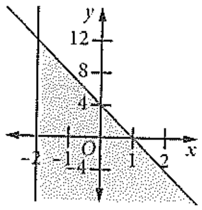
(c) yes, no, yes



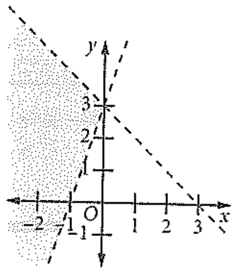
(d) no, yes, yes



(e) yes, no, yes

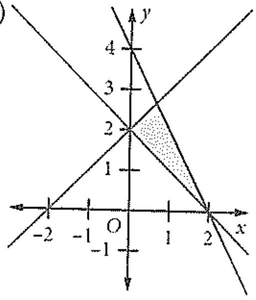


(f) no, no, no



5 (a) correct (b) incorrect (c) correct (d) correct

6 (a)



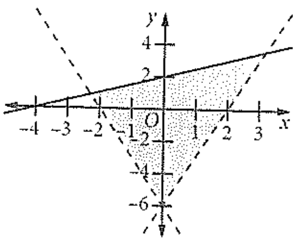
$y = x + 2$, $2x + y = 4$;

$2x + x + 2 = 4$, $3x = 2$:

$x = \frac{2}{3}$, $y = 2\frac{2}{3}$

vertices: $\left(\frac{2}{3}, 2\frac{2}{3}\right)$, $(2, 0)$, $(0, 2)$

(b)



$2y - x = 4$, $y = 3x - 6$;

$6x - 12 - x = 4$, $5x = 16$:

$x = 3\frac{1}{5}$, $y = 3\frac{3}{5}$

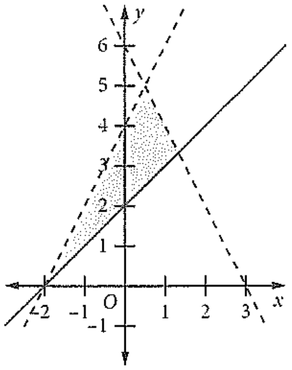
$2y - x = 4$, $3x + y = -6$;

$6x - 12 - x = 4$, $7x = -16$:

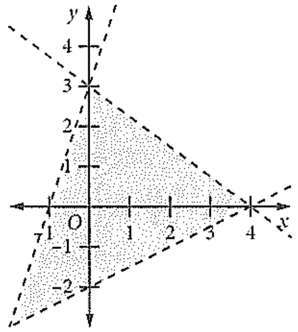
$x = -2\frac{2}{7}$, $y = \frac{6}{7}$

vertices: $\left(3\frac{1}{5}, 3\frac{3}{5}\right)$, $\left(-2\frac{2}{7}, \frac{6}{7}\right)$, $(0, -6)$

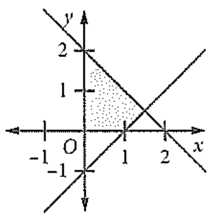
(c) vertices: $(-2,0)$, $(0.5,5)$, $\left(1\frac{1}{3}, 3\frac{1}{3}\right)$



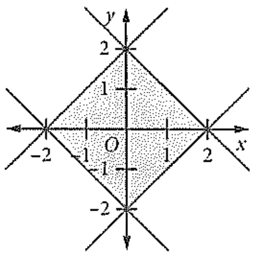
(d) vertices: $(-2,-3)$, $(4,0)$, $(0,3)$



(e) vertices: $(0,0)$, $(1,0)$, $(1.5,0.5)$, $(0,2)$



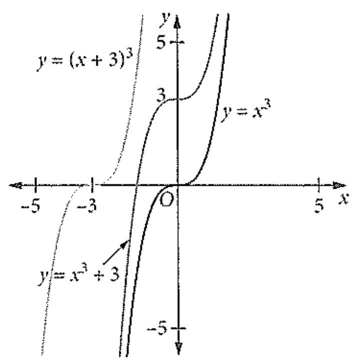
(f) vertices: $(-2,0)$, $(0,2)$, $(2,0)$, $(0,-2)$



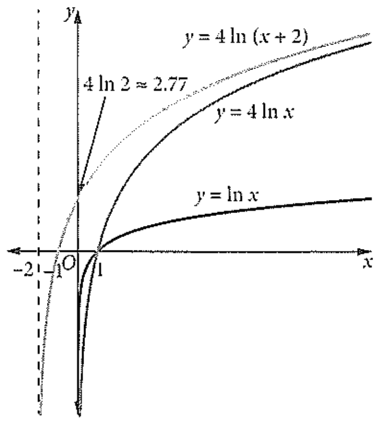
7 (a) the region bounded by the lines $y=x$, $x=2$ and $y=1$; $y \le x$, $x \le 2$, $y \ge 1$; $A(2,2)$, $B(2,1)$, $C(1,1)$
(b) the region bounded by the lines $y=2x$, $y=x$ and $x+y=3$; $y \le 2x$, $y \ge x$, $x+y \le 3$; $A(0,0)$, $B(1,2)$, $C(1.5,1.5)$
(c) the region bounded by the lines $y=\frac{x}{2}+1$, $y=\frac{x}{6}+1$ and $x+2y=6$; $y \le \frac{x}{2}+1$, $y \ge \frac{x}{6}+1$, $x+2y \le 6$; $A(0,1)$, $B(2,2)$, $C(3,1.5)$

CHAPTER REVIEW 15

1

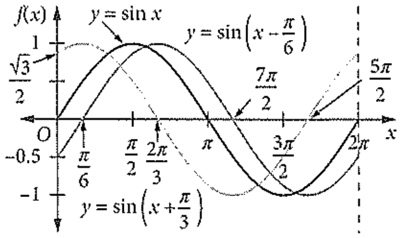


2

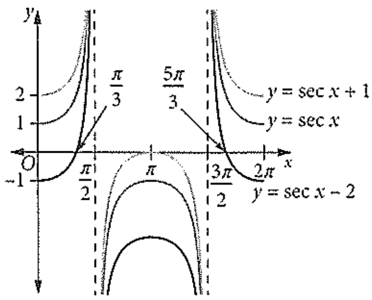


The dilation from the x-axis for the second and third graphs has factor 4.

3 (a)



(b)

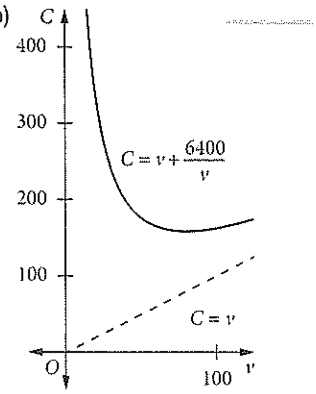


4 (a) $f(2x)=e^{2x}$ (b) $f(x-3)=e^{x-3}$ (c) $f(x)+1=e^x+1$
(d) $2f(x)+4=2e^x+4$ (e) $f(2(x+2))-1=e^{2(x+2)}-1$

5 $C=v+\frac{6400}{v}$

(a) $0 < v \le 110$

(b)



(c) $\frac{dC}{dt}=1-\frac{6400}{v^2}$

$\frac{dC}{dt}=0$: $1-\frac{6400}{v^2}=0$

$v^2=6400$

$v=80$

Average speed is 80 km h^{-1}.

6 (a) Let $BC = y$ cm

$AB = AE = EB = DC = x$ cm

$BC = BF = FC = AD = y$ cm

$3x + 3y = 54$

$x + y = 18$

$y = 18 - x$

$A = xy + \frac{1}{2}x^2 \sin 60° + \frac{1}{2}y^2 \sin 60°$

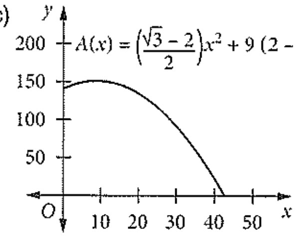
$$A(x) = x(18 - x) + x^2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + (18 - x)^2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= 18x - x^2 + \frac{\sqrt{3}}{4}x^2 + 81\sqrt{3} - 9\sqrt{3}x + \frac{\sqrt{3}}{4}x^2$$

$$= \frac{(\sqrt{3}-2)}{2}x^2 + 9(2-\sqrt{3})x + 81\sqrt{3}$$

(b) The domain is $0 < x < 18$.

(c)



(d) $\frac{dA}{dx} = (\sqrt{3}-2)x + 9(2-\sqrt{3})$

$\frac{dA}{dx} = 0$: $(\sqrt{3}-2)x + 9(2-\sqrt{3}) = 0$

$x = 9$

The rectangle becomes a square of side 9 cm when the area is a maximum.

7 (a) (i) $f(x) = 4 + \frac{3x-1}{x^2}$

$4 + \frac{3x-1}{x^2} = 0$

$4x^2 + 3x - 1 = 0$

$(4x - 1)(x + 1) = 0$

$x = -1, \frac{1}{4}$

(ii) As $x \to \infty, f(x) \to 4$

(iii) $f(x)$ is undefined at $x = 0$, it approaches $-\infty$.

(iv) Asymptotes are $x = 0$ and $y = 4$

(v) $f'(x) = 0 + \frac{3x^2 - (3x-1)\times 2x}{x^4} = \frac{3x - 6x + 2}{x^3} = \frac{2-3x}{x^3}$

$f'(x) = 0$: $x = \frac{2}{3}, y = 6\frac{1}{4}$

$f''(x) = \frac{-3x^3 - (2-3x)\times 3x^2}{x^6} = \frac{-3x - 6 + 9x}{x^4} = \frac{6(x-1)}{x^4}$

$f''\left(\frac{2}{3}\right) = 6 \times \frac{3^4}{2^4} \times \left(-\frac{1}{3}\right) < 0$

$\left(\frac{2}{3}, 6\frac{1}{4}\right)$ is a maximum turning point.

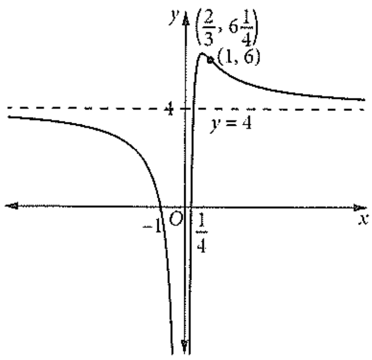
(vi) $f''(x) = 0$ when $x = 1$.

$f''\left(\frac{2}{3}\right) < 0$

$f''(2) = 6 \times \frac{1}{16} > 0$

Concavity changes at $x = 1$ so $(1, 6)$ is a point of inflection.

(b)



8 (a) If $y = \frac{4x}{(x-1)^2}$

(i) $\frac{dy}{dx} = \frac{4(x-1)^2 - 4x \times 2(x-1)}{(x-1)^4} = \frac{-4(x+1)}{(x-1)^3}$

For stationary points, $\frac{dy}{dx} = 0$: $x = -1, y = -1$

$$\frac{d^2y}{dx^2} = \frac{-4\left((x-1)^3 - (x+1)\times 3(x-1)^2\right)}{(x-1)^6}$$

$$= \frac{-4(x - 1 - 3x - 3)}{(x-1)^4} = \frac{8(x+2)}{(x-1)^4}$$

$x = -1$: $\frac{d^2y}{dx^2} = \frac{8}{16} > 0$

Minimum turning point at $(-1, -1)$.

(ii) $\frac{d^2y}{dx^2} = 0$: $x = -2, y = -\frac{8}{9}$.

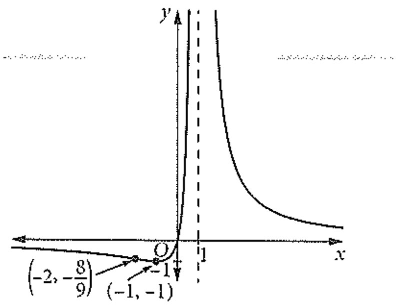
$x = -1$: $\frac{d^2y}{dx^2} > 0$

$x = -3$: $\frac{d^2y}{dx^2} = \frac{-8}{4^4} < 0$

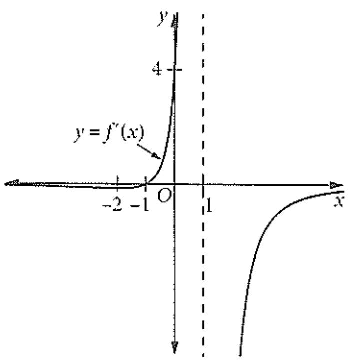
Concavity changes at $x = -2$ so $\left(-2, \frac{-8}{9}\right)$ is a point of inflection.

(iii) $x = 1, y = 0$

(b)



(c) $f'(x) = \frac{-4(x+1)}{(x-1)^3}$



9 (a) $\dfrac{x(x+1)}{x-1} = \dfrac{x^2+x}{x-1}$, $px + q + \dfrac{r}{x-1} = \dfrac{px^2+(q-p)x+(r-q)}{x-1}$,

$p = 1, q = 2, r = 2$;

$\dfrac{x(x+1)}{x-1} = x + 2 + \dfrac{2}{x-1}$

(b) $y = \dfrac{x(x+1)}{x-1} = x + 2 + \dfrac{2}{x-1}$,

$\dfrac{dy}{dx} = 1 - \dfrac{2}{(x-1)^2}$

$\dfrac{dy}{dx} = 0$: $(x-1)^2 = 2, x - 1 = \pm\sqrt{2}, x = 1 \pm \sqrt{2}$

$x = 1 + \sqrt{2}, y = 1 + \sqrt{2} + 2 + \dfrac{2}{\sqrt{2}} = 3 + 2\sqrt{2}$

$x = 1 - \sqrt{2}, y = 1 - \sqrt{2} + 2 + \dfrac{2}{-\sqrt{2}} = 3 - 2\sqrt{2}$

$\dfrac{d^2y}{dx^2} = \dfrac{4}{(x-1)}$

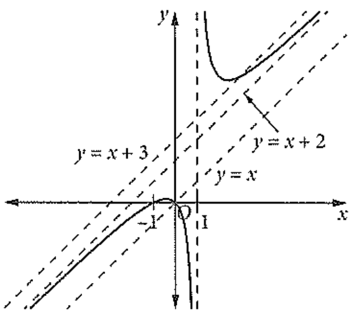
$x = 1 + \sqrt{2}$: $\dfrac{d^2y}{dx^2} = \dfrac{4}{2\sqrt{2}} > 0$

Minimum turning point at $(1 + \sqrt{2}, 3 + 2\sqrt{2})$

$x = 1 - \sqrt{2}$: $\dfrac{d^2y}{dx^2} = \dfrac{4}{-2\sqrt{2}} < 0$

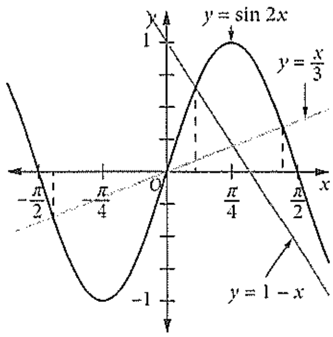
Maximum turning point at $(1 - \sqrt{2}, 3 - 2\sqrt{2})$

(c), (d)



(e) $c = 2$. $y = x + 2$ is the sloping asymptote.

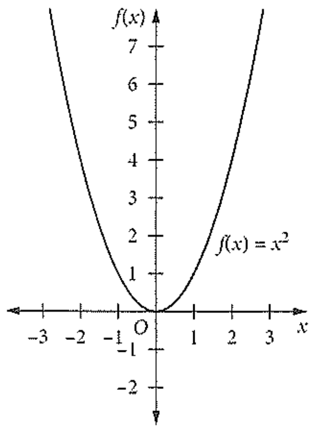
10

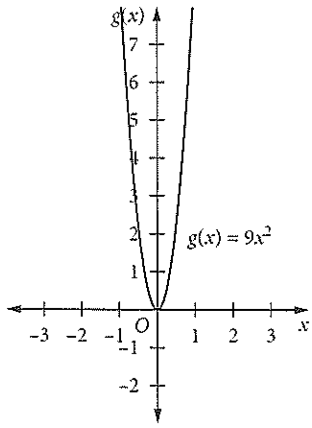


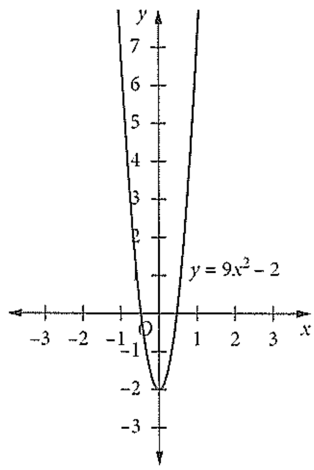
(a) The graphs $y = \sin 2x$ and $y = \dfrac{x}{3}$ intersect at three places; when $x = 0$ and near $x = \pm 0.4\pi$ or ± 1.3. More accurate solutions using technology give $x = 0, \pm 0.43\pi$ or ± 1.34.

(b) The graphs $y = \sin 2x$ and $y = 1 - x$ intersect at one place: near $x = 0.1\pi$ or 0.3. A more accurate solution using technology gives $x = 0.11\pi$ or 0.35.

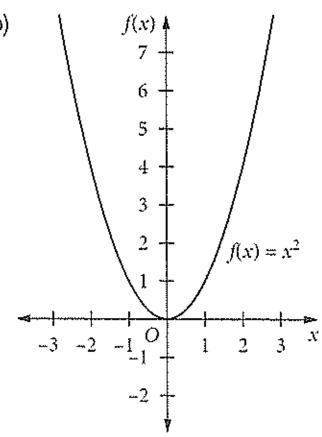
11 (a)

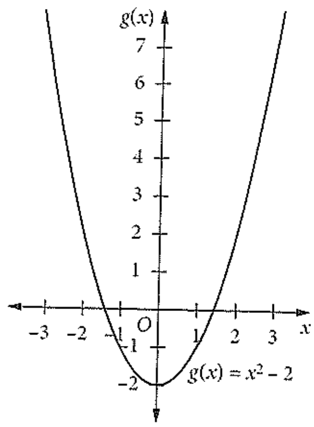


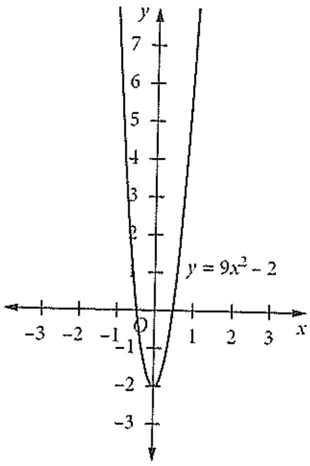




(b)

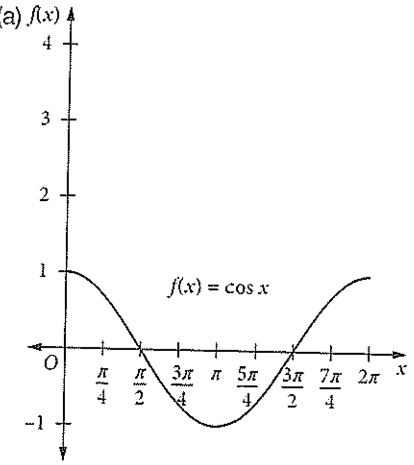


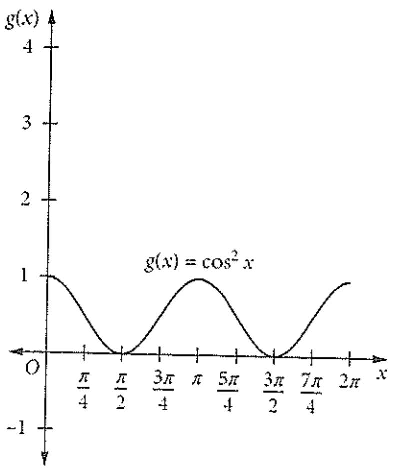


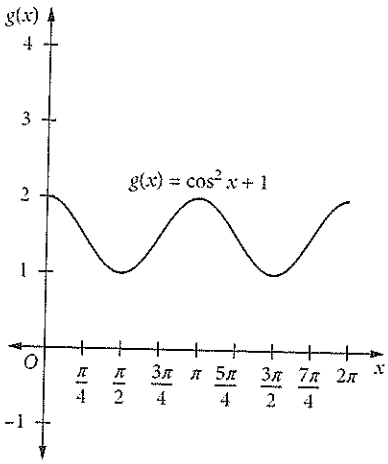


(c) The graphs are the same.

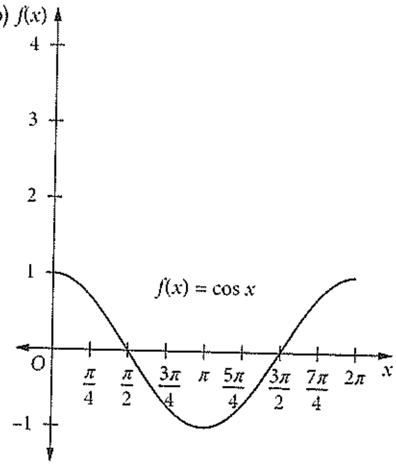
12 (a)

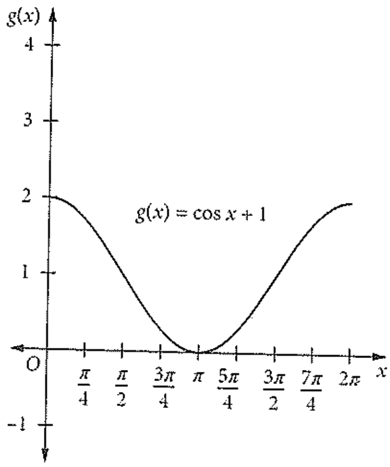


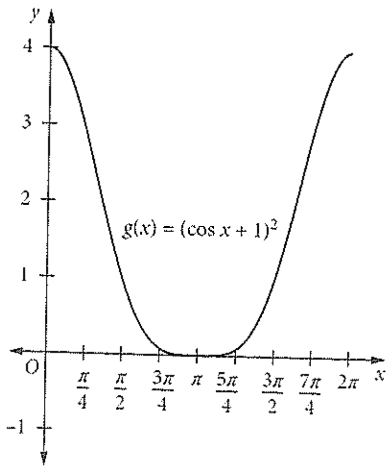




(b)

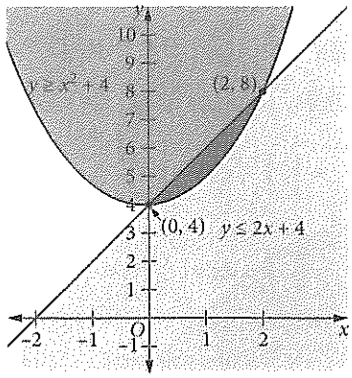






(c) The final graphs in each part are completely different due to the time the squaring is done.

13 (a)



(b) $0 \leq x \leq 2$

CHAPTER 16

EXERCISE 16.1

1 (a) $2x^3 - 2x^2 + 5x + C$ (b) $3x + \frac{5x^2}{2} + \frac{x^3}{3} - \frac{3x^4}{4} + C$
(c) $\frac{x^3}{3} - x + C$ (d) $\frac{x^6}{6} - x^4 - \frac{2x^3}{3} + x + C$ (e) $\frac{3x^2}{2} - 5x + C$
(f) $\frac{2x^3}{3} - \frac{7x^2}{2} + 5x + C$

2 (a) correct (b) correct (c) incorrect (d) incorrect

3 (a) $\frac{4x^3}{3} + 2x^2 + x + C$ (b) $5x + C$ (c) $\frac{x^3}{3} + \frac{3x^2}{2} + C$
(d) $\frac{2x^3}{3} + \frac{3x^2}{2} - 2x + C$ (e) $x^4 - 2x^3 + \frac{x^2}{2} + C$
(f) $\frac{x^3}{3} + 3x^2 + 9x + C$

4 $\text{LHS} = \frac{8x^3 + 12x^2 + 6x + 1}{6} = \frac{4x^3}{2} + 2x^2 + x + 1$,
which differs from RHS only by a constant

5 (a) $3x + x^2 - x^3 + C$ (b) $\frac{x^4}{4} + \frac{2x^3}{3} + C$ (c) $\frac{x^5}{5} - \frac{x^4}{4} + C$
(d) $\frac{x^3}{3} + \frac{x^2}{2} - 12x + C$ (e) $\frac{x^5}{5} - \frac{x^4}{12} + \frac{x^3}{6} + C$ (f) $\frac{2x^{\frac{3}{2}}}{3} + C$

6 (a) $\frac{x^4}{4} - x^3 + C$ (b) $\frac{x^3}{15} - \frac{x^4}{16} + C$ (c) $\frac{x^3}{3} - \frac{3x^4}{4} + C$
(d) $\frac{x^5}{5} - x + C$ (e) $-2x^{\frac{-1}{2}} - \frac{2x^{\frac{-3}{2}}}{3} + C$ (f) $\frac{2x^{\frac{3}{2}}}{3} + \frac{3x^{\frac{4}{3}}}{4} + C$

7 $x^2 - 2x + 5$ 8 C 9 $y = 2x^2 - 6x + 8$

10 $y = x^3 - x^2 + 3x - 24$ 11 $F(x) = \frac{(x+2)^3}{3} - 5$ 12 $y = x^2 - 4x$

13 $f(x) = \frac{x^3}{3} - x^2 + x + 2$ 14 $y = \frac{x^3}{3} + \frac{x^2}{2} - 2x + 4$

15 (a) $3x - \frac{1}{x} + \frac{1}{x^2} + C$ (b) $\frac{x^2 - 2}{x} + C = x - \frac{2}{x} + C$
(c) $\frac{3x^2}{2} - 2x - \frac{1}{2x^2} + C$ (d) $C - \frac{1}{2x^2}$ (e) $\frac{4x^{\frac{3}{2}}}{3} + 3x - \frac{5}{x} + C$
(f) $2\sqrt{x} - \frac{2}{\sqrt{x}} + C$

16 $s = 4t^3 - 3t^2 + t + 2$ 17 $N = 60t + 50,410$ 18 $d = t^3 + 4t$

EXERCISE 16.2

1 (a) $\frac{x^2}{2} + C$ (b) $\frac{x^3}{3} + \frac{x^2}{2} + x + C$ (c) $3x - \frac{x^3}{3} + C$
(d) $x^6 - x^4 + x^2 + C$ (e) $x + C$ (f) $\frac{x^{n+1}}{n+1} + C$

2 (a) $\frac{2}{3}x^{\frac{3}{2}} + C$ (b) $C - \frac{1}{x}$ (c) $x + \frac{2x\sqrt{x}}{3} + \frac{x^2}{2} + C$
(d) $\frac{x^2}{2} - \frac{1}{x} + C$ (e) $\frac{x^3}{3} + 2x - \frac{1}{x} + C$ (f) $x - \frac{4}{3}x^{\frac{3}{2}} + \frac{x^2}{2} + C$

3 C 4 $y = x + \frac{x^2}{2} + x^3 - 6$

5 $y = x + \frac{2x\sqrt{x}}{3} + \frac{2}{3}$ OR $y = x + \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{3}$

EXERCISE 16.3

1 (a) $C - \frac{\cos 2x}{2}$ (b) $\frac{\sin 3x}{3} + C$ (c) $\tan x + C$
(d) $\sin x - \cos x + C$ (e) $-2\cos x - 3\sin x + C$
(f) $C - \cos\left(x + \frac{\pi}{4}\right)$ (g) $2\sin\frac{x}{2} + C$ (h) $C - \cos 2x$

2 D

3 (a) $4\sin\frac{x}{4} - 4\cos\frac{x}{4} + C$ (b) $C - \cos x - \frac{\sin 2x}{2}$
(c) $-\frac{1}{2}\cos\left(2x + \frac{\pi}{2}\right) + C = \frac{\sin 2x}{2} + C$ (d) $\frac{1}{2}\sin\left(2x - \frac{\pi}{4}\right) + C$
(e) $\frac{\tan 3x}{3} + C$ (f) $C - \frac{1}{4}\cos 2x - \sin x$
(g) $2\sin\frac{x}{2} + \frac{1}{4}\cos 2x + C$ (h) $\frac{x^3}{3} - \frac{\cos 2x}{2} + C$
(i) $\frac{2}{3}x^{\frac{3}{2}} - \frac{\sin x}{2} + C$

4 (a) $\ln(1 - \cos x) + C$ (b) $\ln(\sin x) + C$

5 $y = \frac{7 - 2\cos 3x}{3}$

EXERCISE 16.4

1 (a) $\frac{e^{2x}}{2}$ (b) $\frac{e^{5x}}{5}$ (c) $\frac{-5e^{-0.4x}}{2}$
(d) $2e^{2.5x}$ (e) $e^x - \frac{e^{-3x}}{3}$ (f) $e^{-x} - \frac{e^{-2x}}{2}$

2 (a) $-e^{-x} + C$ (b) $2e^{\frac{x}{2}} + C$ (c) $C - \frac{e^{-3x}}{3}$
(d) $-e^{-t} - t + C$ (e) $\frac{e^{2u}}{2} + \frac{u^3}{3} + C$ (f) $\frac{-2e^{-2.5}}{5} + \frac{5e^{0.4x}}{2} + C$

EXERCISE 16.5

1 (a) $2\ln x + C$ (b) $\ln(x+1) + C$ (c) $\ln(2x+1) + C$
(d) $\frac{1}{2}\ln\left(x^2 - 4\right) + C$ (e) $\frac{1}{2}\ln(2x-1) + C$ (f) $\ln\left(4 + e^x\right) + C$
(g) $\frac{1}{4}\ln\left(x^4 + 1\right) + C$ (h) $-\frac{1}{2}\ln\left(4 - e^{2x}\right) + C$

2 C 3 $y = \log_e(2x+1)$

4 (a) correct (b) correct (c) incorrect (d) correct

5 $\frac{d}{dx}(\log_e(\cos x)) = \frac{-\sin x}{\cos x} = -\tan x$, $\int \tan x \, dx = C - \log_e(\cos x)$
area $= \int_0^{\frac{\pi}{4}} \tan x \, dx = \left[-\log_e(\cos x)\right]_0^{\frac{\pi}{4}} = -\log_e\frac{1}{2} + \log_e 1 = \log_e 2$

6 (a) $2^x \ln 2$ (b) $1 + 10^x \ln 10$ (c) $e^x + 5^x \ln 5$
(d) $\frac{d}{dx}\left(5^{x^2}\right) = 5^{x^2} \ln 5 \times 2x = 2x \times 5^{x^2} \ln 5$
(e) $\frac{d}{dx}\left(a^{\sqrt{x}}\right) = a^{\sqrt{x}} \ln a \times \frac{1}{2\sqrt{x}} = \frac{a^{\sqrt{x}} \ln a}{2\sqrt{x}}$

7 (a) $\frac{3^x}{\ln 3} + C$ (b) $\frac{x^2}{2} + \frac{10^x}{\ln 10} + C$ (c) $\ln|x| + x + e^x + \frac{a^x}{\ln a} + C$

CHAPTER REVIEW 16

1 (a) $\frac{x^2}{2} + 9x + C$ (b) $x^3 - x^2 + 4x + C$ (c) $\frac{x^5}{5} + \frac{x^4}{4} - 2x + C$
(d) $\frac{x^3}{3} + \frac{x^2}{2} - 6x + C$ (e) $\frac{(x+2)^3}{3} + C = \frac{x^3}{3} + 2x^2 + 4x + C$
(f) $7x + C$

2 (a) $\frac{5x^2}{2} + 4x + C$ (b) $5x - 2x^2 + x^3 + \frac{x^4}{4} + C$
(c) $x^2 + \frac{2x\sqrt{x}}{3} + 3x + C$