

9 $a = 5, r = 4$

a Step 3: Prove true for $n = k + 1$.

$$\begin{aligned} &\text{Prove } a + ar + ar^2 + \dots + ar^{k-1} + ar^k \\ &= \frac{a(r^{k+1} - 1)}{r - 1} \end{aligned}$$

$$\begin{aligned} \text{LHS} &= a + ar + ar^2 + \dots + ar^{k-1} + ar^k \\ &= \frac{a(r^k - 1)}{r - 1} + ar^k \end{aligned}$$

b Step 3: Prove true for $n = k + 1$.

$$\text{Prove } 1 + x + x^2 + \dots + x^{k-1} + x^k = \frac{1 - x^{k+1}}{1 - x}$$

$$\begin{aligned} \text{LHS} &= 1 + x + x^2 + \dots + x^{k-1} + x^k \\ &= \frac{1 - x^k}{1 - x} + x^k \end{aligned}$$

10 a $\operatorname{cosec}^2 x$

b $r = \cos^2 x$

$$-1 < \cos x < 1 \text{ where } \cos^2 x \neq 0, 1$$

$$\text{So } 0 < \cos^2 x < 1$$

Since $|r| < 1$, the series has a limiting sum.

Chapter 2

Exercise 2.01

1 a Vertical translation 3 units up

b Vertical translation 7 units down

c Vertical translation 1 unit down

d Vertical translation 5 units up

2 a Vertical translation 1 unit up

b Vertical translation 4 units down

c Vertical translation 8 units up

3 Vertical translation 9 units up

4 a $y = x^2 - 3$ b $f(x) = 2^x + 8$

c $y = |x| + 1$ d $y = x^3 - 4$

e $f(x) = \log x + 3$ f $y = \frac{2}{x} - 7$

5 a Vertical translation 1 unit down

b Vertical translation 6 units up

6 a i $y = 2x^3 - 2$ ii $y = 2x^3 + 6$

b i $y = |x| - 3$ ii $y = |x| - 6$

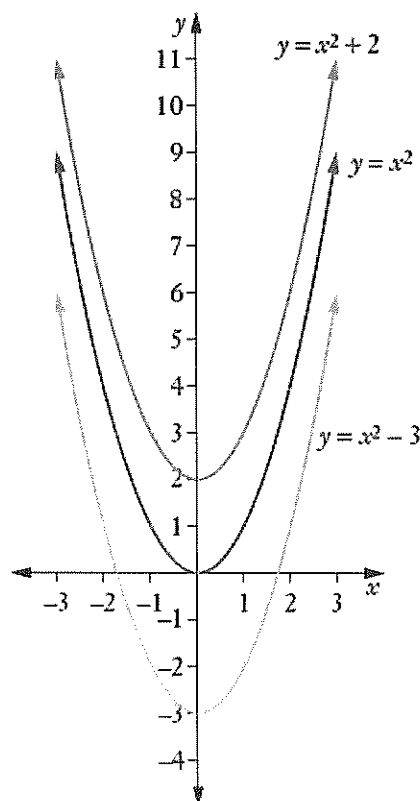
c i $y = e^x + 1$ ii $y = e^x + 5$

d i $f(x) = \log_e x + 10$ ii $f(x) = \log_e x - 8$

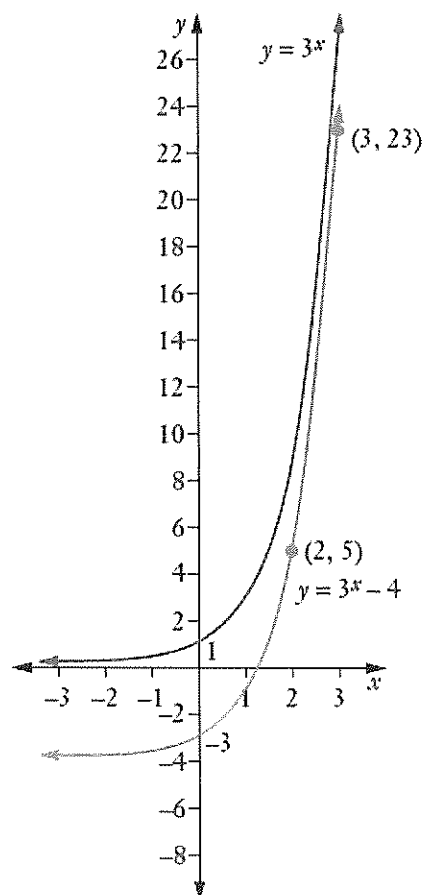
7 a $(1, -1)$ b $(1, -9)$ c $(1, -3 + m)$

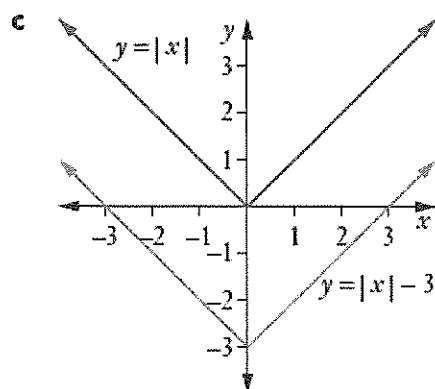
8 a $(-1, 1)$ b $(-1, 5)$

9 a

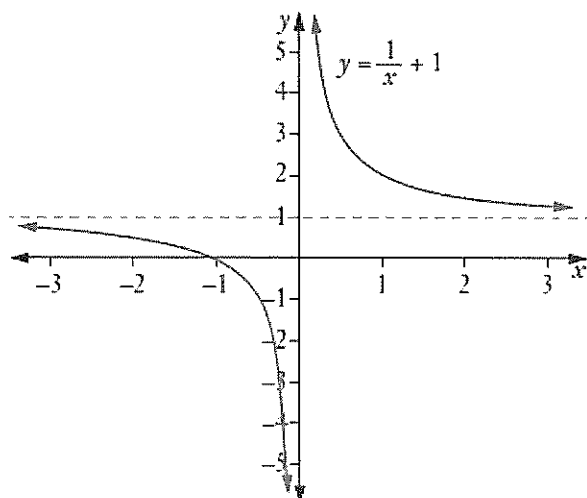


b

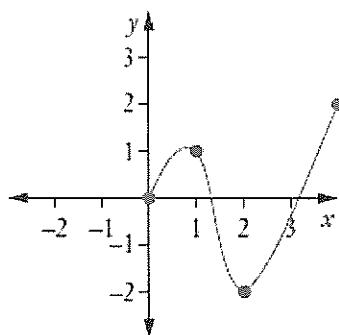




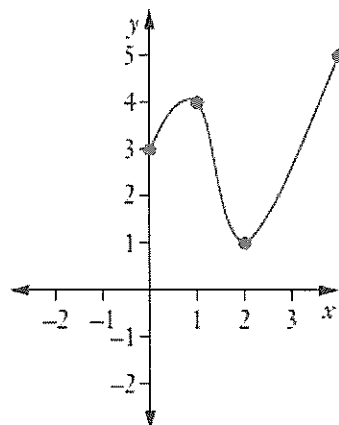
- 10 a** Vertical translation 1 unit up
b



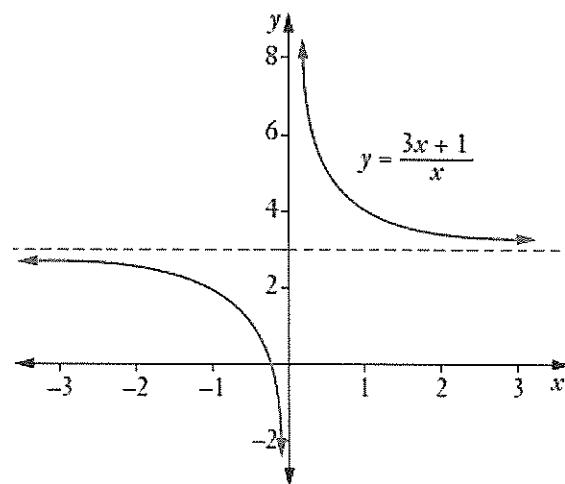
11 a



b



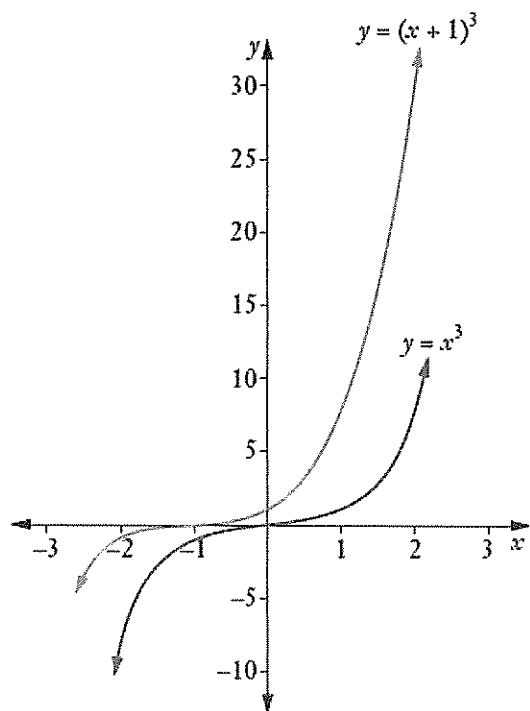
12 b



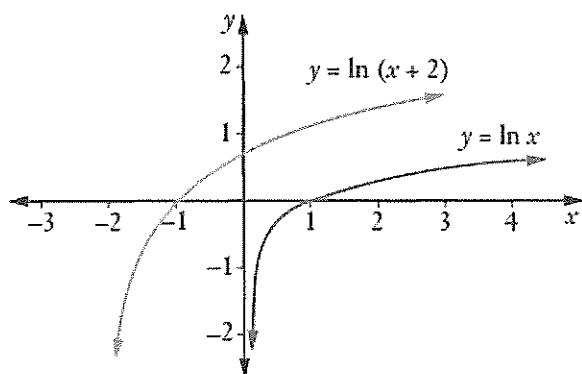
Exercise 2.02

- 1 a** Horizontal translation 4 units to the right
b Horizontal translation 2 units to the left
- 2 a** Horizontal translation 5 units to the right
b Horizontal translation 3 units to the left
- 3 a** $y = (x + 3)^2$ **b** $f(x) = 2^{x-8}$
c $y = |x + 1|$ **d** $y = (x - 4)^3$
e $f(x) = \log(x + 3)$
- 4** Horizontal translation 3 units to the right
- 5 a** Horizontal translation 2 units to the left
b Horizontal translation 5 units to the right
- 6 a i** $y = -(x + 4)^2$ **ii** $y = -(x - 8)^2$
b i $y = |x - 3|$ **ii** $y = |x + 4|$
c i $y = e^{x+6}$ **ii** $y = e^{x-5}$
d i $f(x) = \log_2(x - 5)$ **ii** $f(x) = \log_2 x$
- 7 a** $(5, -3)$ **b** $(-8, -3)$ **c** $(1 + t, -3)$
- 8 a** $(3, 2)$ **b** $(-9, 2)$

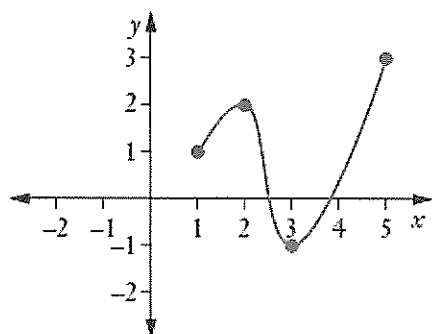
9 a



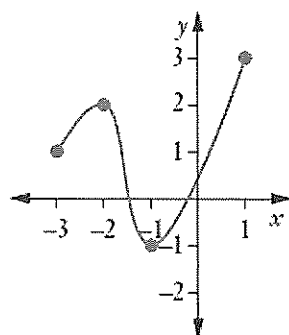
b



10 a



b



11 a $f(x) = x^5 - 5$

b $f(x) = (x - 3)^5$

c $f(x) = x^5 + 2$

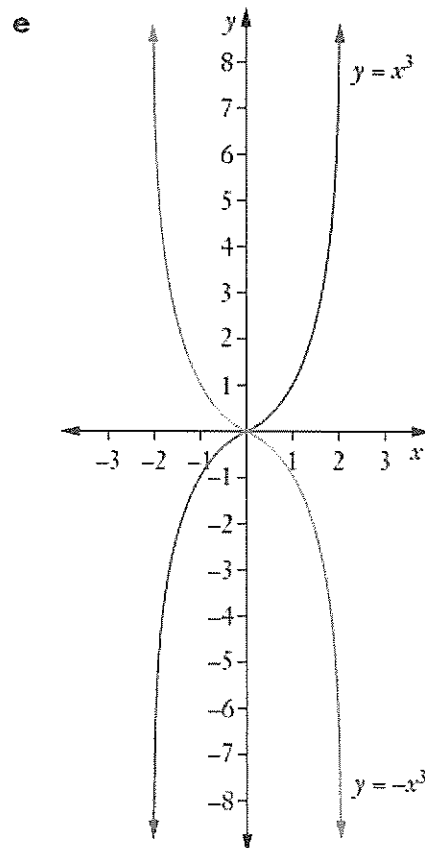
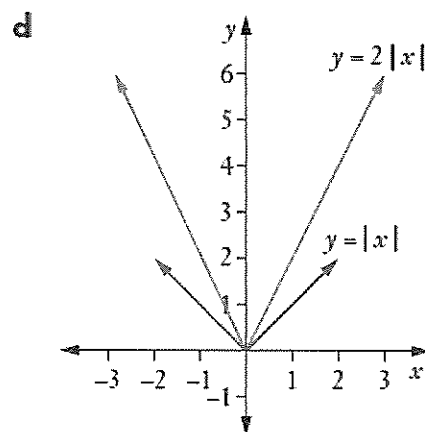
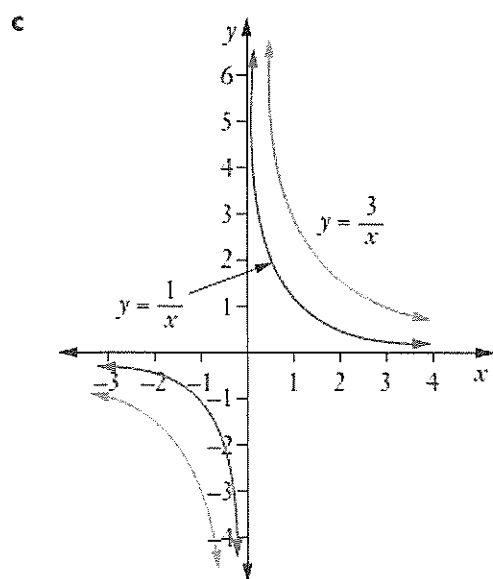
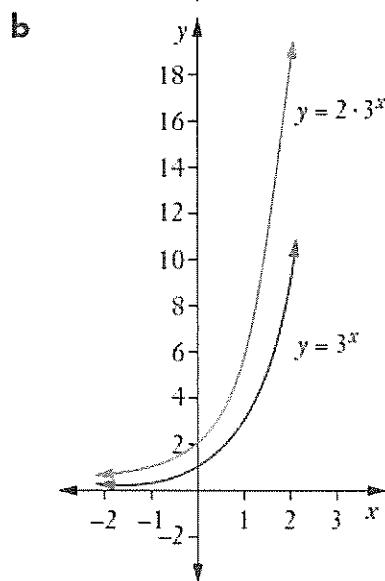
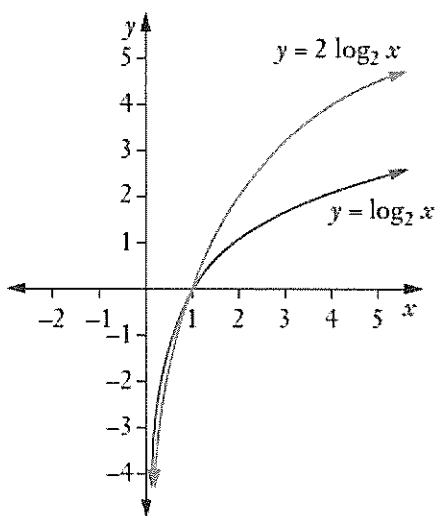
d $f(x) = (x + 7)^5$

12 $(7, -2)$

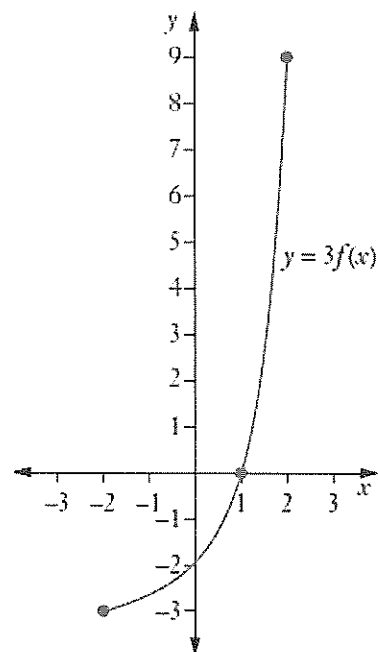
Exercise 2.03

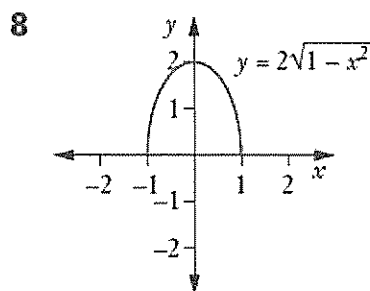
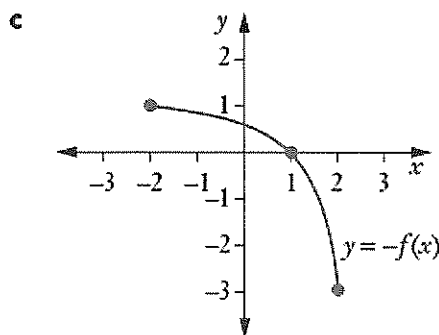
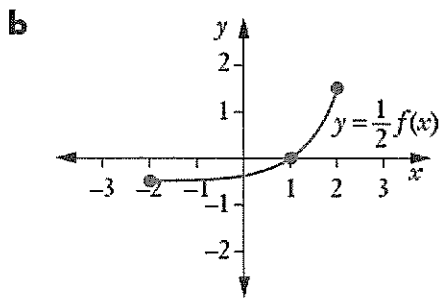
- 1 a
 - i Vertical dilation scale factor 6 (stretched)
 - ii Vertical dilation scale factor $\frac{1}{2}$ (compressed)
 - iii Vertical dilation scale factor -1 (reflection in x -axis)
 - b
 - i Vertical dilation scale factor 2 (stretched)
 - ii Vertical dilation scale factor $\frac{1}{6}$ (compressed)
 - iii Vertical dilation scale factor -1 (reflection in x -axis)
 - c
 - i Vertical dilation scale factor 4 (stretched)
 - ii Vertical dilation scale factor $\frac{1}{7}$ (compressed)
 - iii Vertical dilation scale factor $\frac{4}{3}$ (stretched)
 - d
 - i Vertical dilation scale factor 9 (stretched)
 - ii Vertical dilation scale factor $\frac{1}{3}$ (compressed)
 - iii Vertical dilation scale factor $\frac{3}{8}$ (compressed)
 - e
 - i Vertical dilation scale factor 5 (stretched)
 - ii Vertical dilation scale factor $\frac{1}{8}$ (compressed)
 - iii Vertical dilation scale factor -1 (reflection in x -axis)
 - f
 - i Vertical dilation scale factor 9 (stretched)
 - ii Vertical dilation scale factor -1 (reflection in x -axis)
 - iii Vertical dilation scale factor $\frac{2}{5}$ (compressed)
- 2 a $y = 6x^2$; domain $(-\infty, \infty)$; range $[0, \infty)$
 - b $y = \frac{\ln x}{4}$; domain $(0, \infty)$; range $(-\infty, \infty)$
 - c $f(x) = -|x|$; domain $(-\infty, \infty)$; range $(-\infty, 0]$
 - d $f(x) = 4e^x$; domain $(-\infty, \infty)$; range $(0, \infty)$
 - e $y = \frac{7}{x}$; domain $(-\infty, 0) \cup (0, \infty)$; range $(-\infty, 0) \cup (0, \infty)$
- 3 a $y = 5 \cdot 3^x$ b $f(x) = \frac{x^2}{3}$ c $y = -x^3$
 - d $y = \frac{1}{2x}$ e $y = \frac{2|x|}{3}$

- 4 **a** (3, 24) **b** (3, -6) **c** (3, 72) **d** (3, 5)
 5 **a** (4, 4) **b** (4, 6) **c** (4, 36) **d** (4, 16)
e (4, -12)
 6 **a**



7 **a**



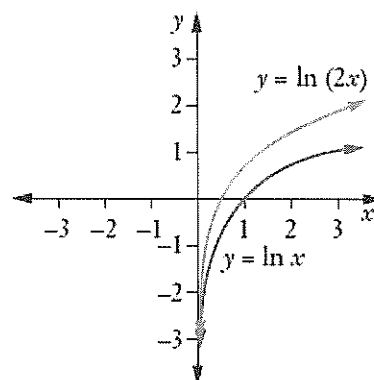


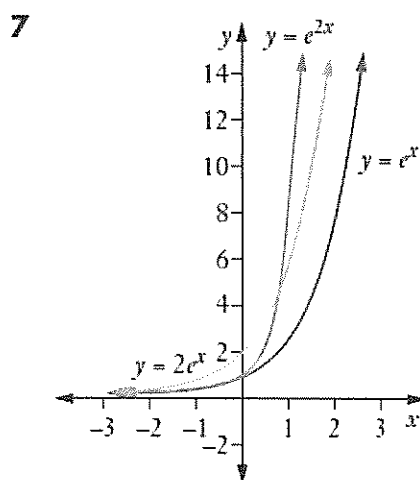
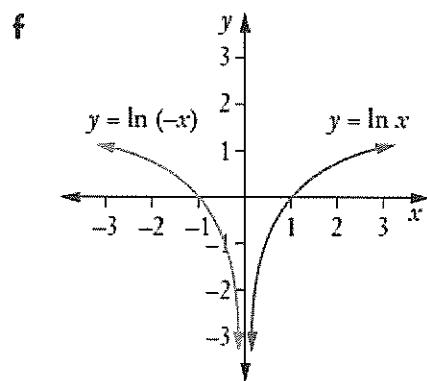
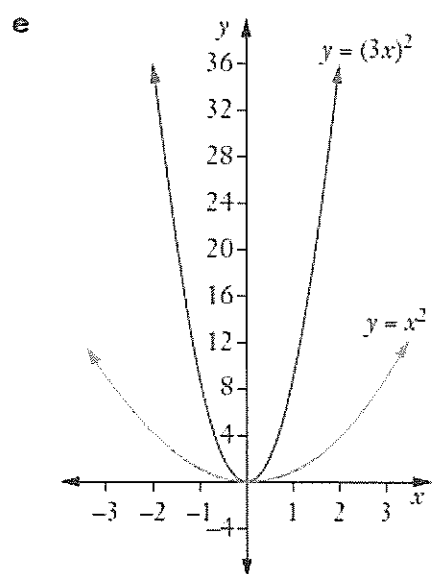
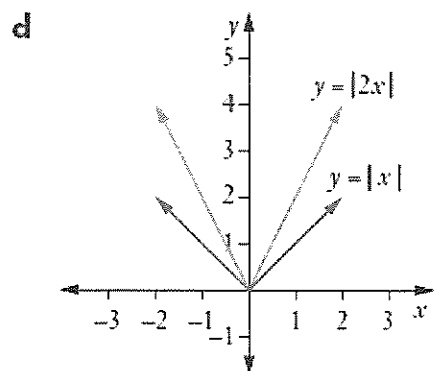
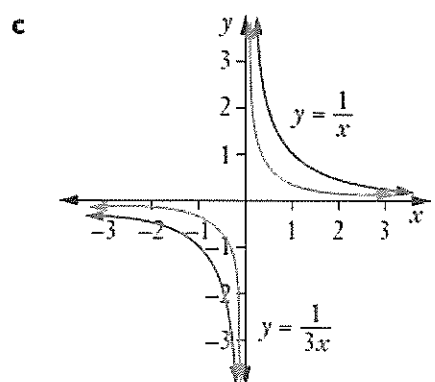
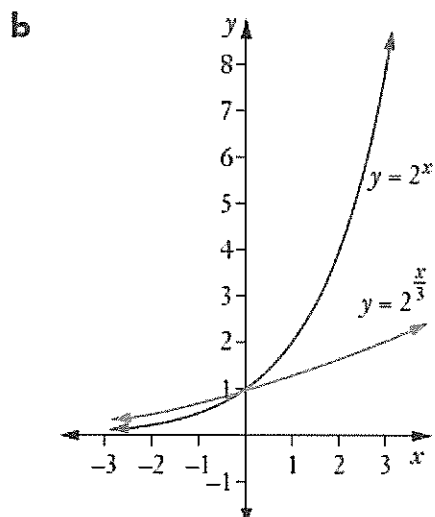
Exercise 2.04

- 1 a** Horizontal dilation scale factor $\frac{1}{8}$ (compressed)
b Horizontal dilation scale factor 5 (stretched)
c Horizontal dilation scale factor $\frac{7}{3}$ (stretched)
d Horizontal dilation scale factor -1 (reflection in y -axis). Note that this is the same graph as $f(x) = x^4$.
- 2 a** **i** Horizontal dilation scale factor $\frac{1}{2}$ (compressed)
ii Horizontal dilation scale factor $\frac{1}{5}$ (compressed)
iii Horizontal dilation scale factor 3 (stretched)
- b** **i** Vertical dilation scale factor 4 (stretched)
ii Horizontal dilation scale factor 2 (stretched)
iii Horizontal dilation scale factor -1 (reflection in y -axis)

- c** **i** Horizontal dilation scale factor $\frac{1}{7}$ (compressed)
ii Vertical dilation scale factor $\frac{1}{8}$ (compressed)
iii Horizontal dilation scale factor $\frac{4}{3}$ (stretched)
- d** **i** Horizontal dilation scale factor $\frac{1}{5}$ (compressed) (or vertical dilation scale factor 5, stretched)
ii Horizontal dilation scale factor 2 (stretched) (or vertical dilation scale factor $\frac{1}{2}$, compressed)
iii Horizontal dilation scale factor $\frac{5}{3}$ (stretched) (or vertical dilation scale factor $\frac{3}{5}$, compressed)
- e** **i** Horizontal dilation scale factor $\frac{1}{3}$ (compressed)
ii Vertical dilation scale factor -1 (reflection in x -axis)
iii Horizontal dilation scale factor 2 (stretched)
- f** **i** Vertical dilation scale factor 8 (stretched)
ii Horizontal dilation scale factor -1 (reflection in y -axis)
iii Horizontal dilation scale factor 7 (stretched)

- 3 a** $f(x) = |5x|$; domain $(-\infty, \infty)$; range $[0, \infty)$
b $y = \left(\frac{x}{3}\right)^2$; domain $(-\infty, \infty)$; range $[0, \infty)$
c $y = (-x)^3$; domain $(-\infty, \infty)$; range $(-\infty, \infty)$
d $y = \frac{e^x}{9}$; domain $(-\infty, \infty)$; range $(0, \infty)$
e $y = -\log_4 x$; domain $(0, \infty)$; range $(-\infty, \infty)$
- 4 a** $(-1, 7)$ **b** $(2, 7)$ **c** $(-6, 7)$
- 5 a** $(-72, 1)$ **b** $(-48, 1)$ **c** $(-6, 1)$
- 6 a**

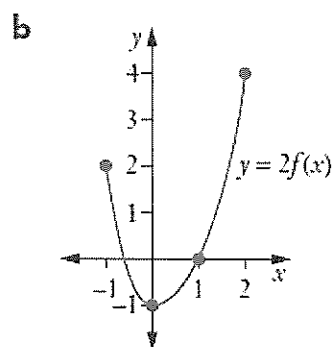
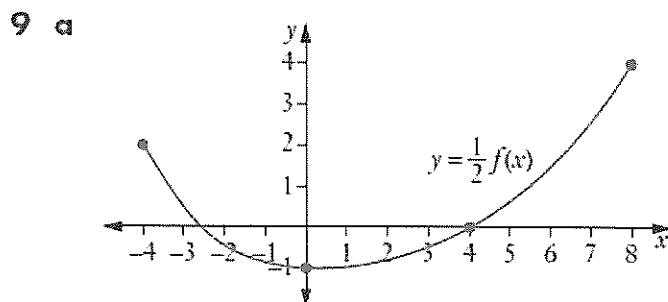




8 A reflection in y -axis transforms $y = f(x)$ into $y = f(-x)$.

a Since $y = x^2$ is an even function, $f(x) = f(-x)$, so a reflection in the y -axis doesn't change the function.

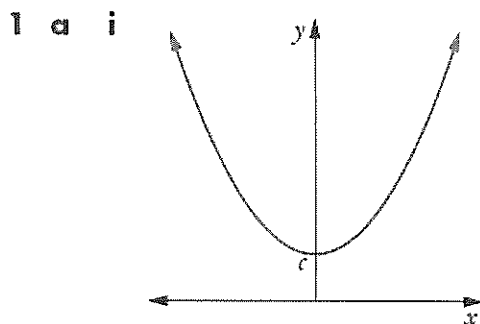
b Since $y = |x|$ is an even function, $f(x) = f(-x)$ so a reflection in the y -axis doesn't change the function.

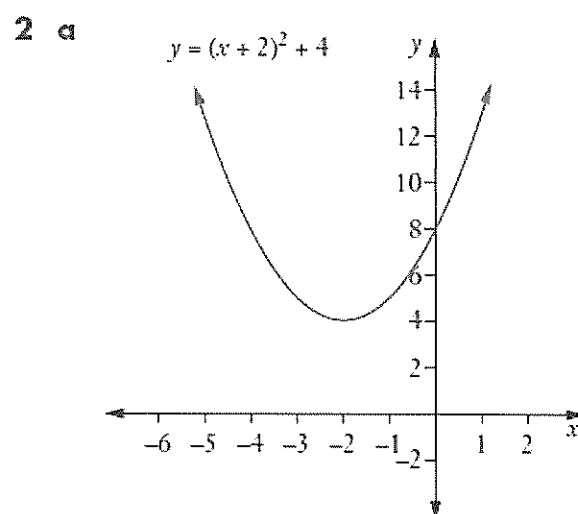
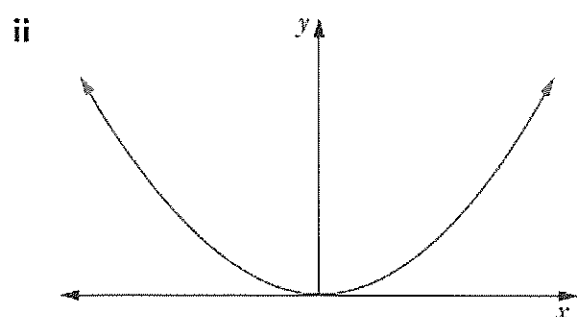
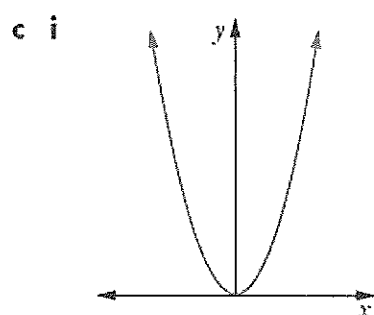
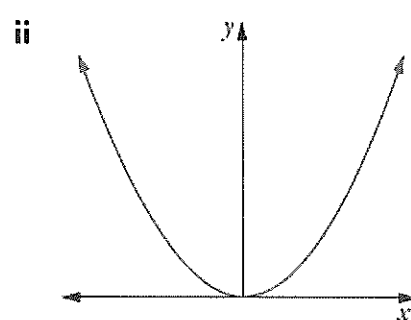
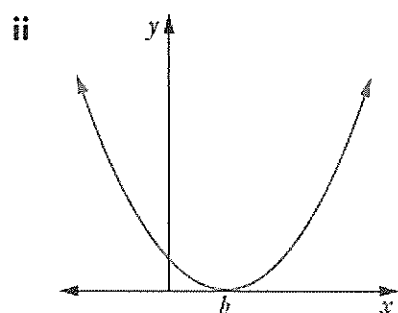
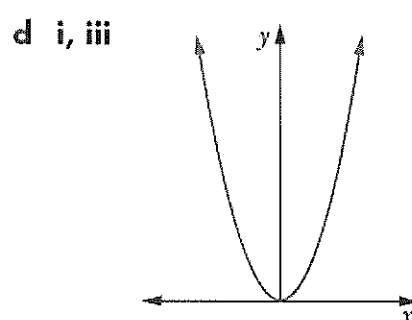
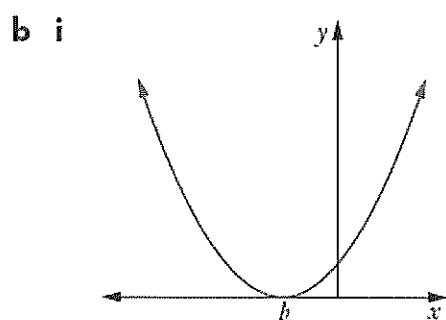
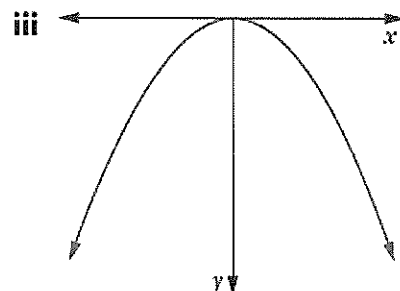
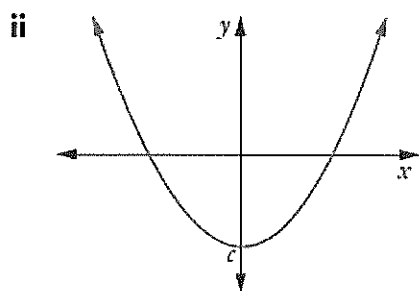


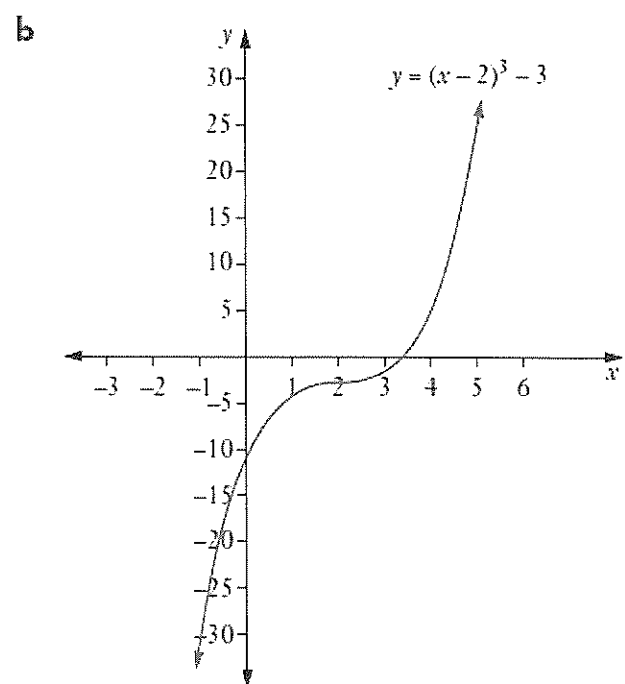
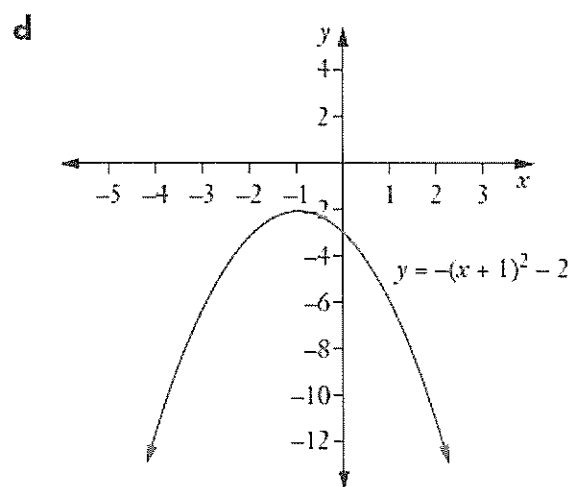
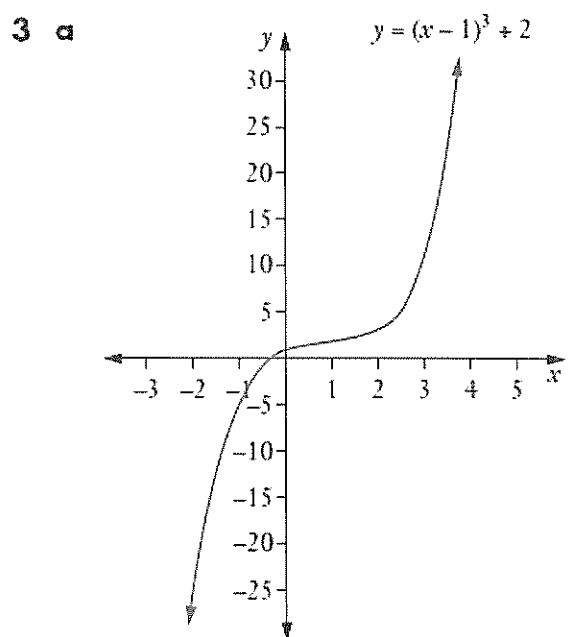
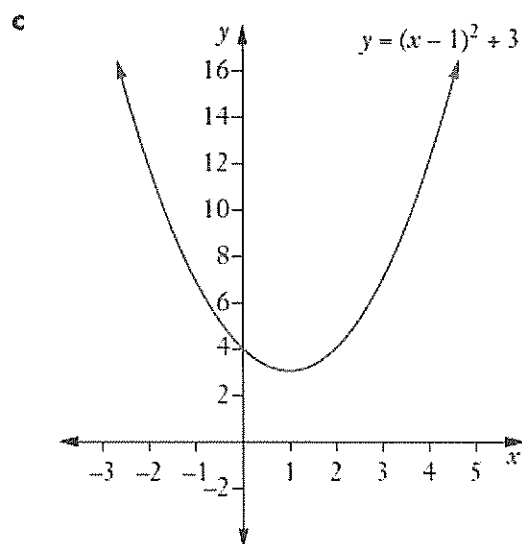
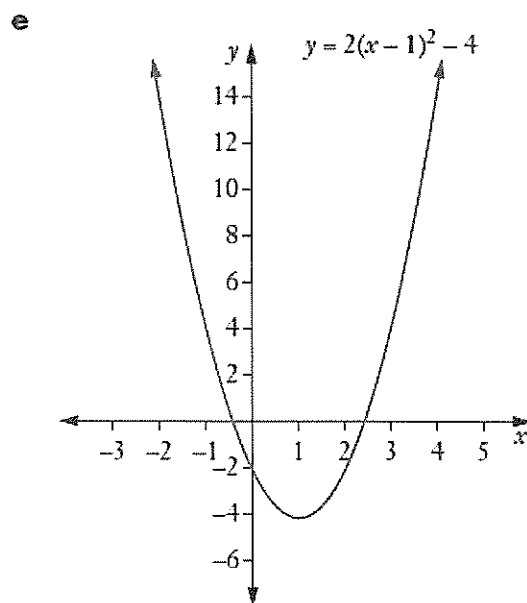
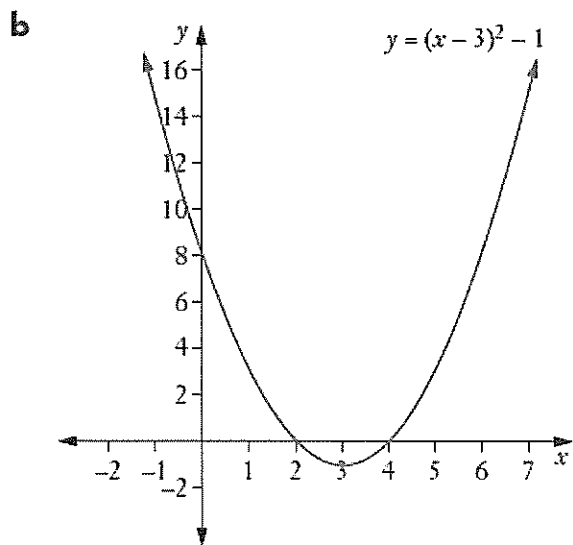
Exercise 2.05

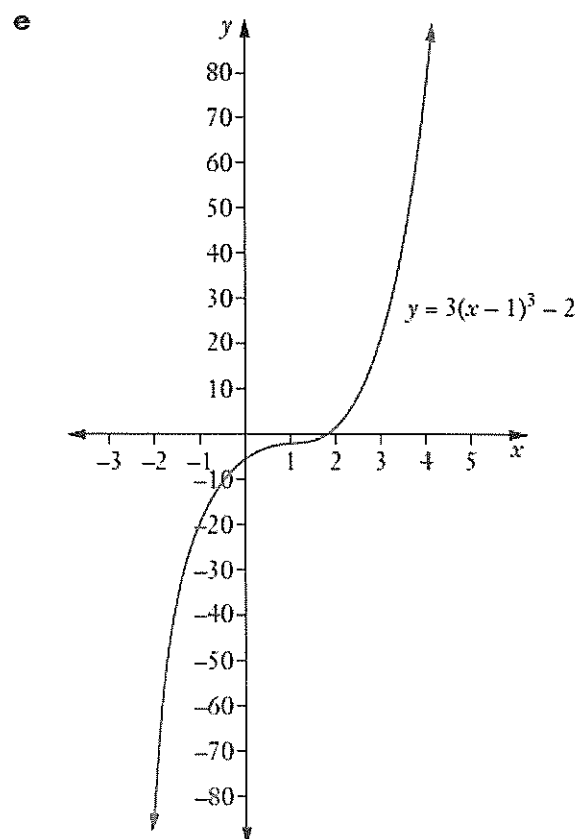
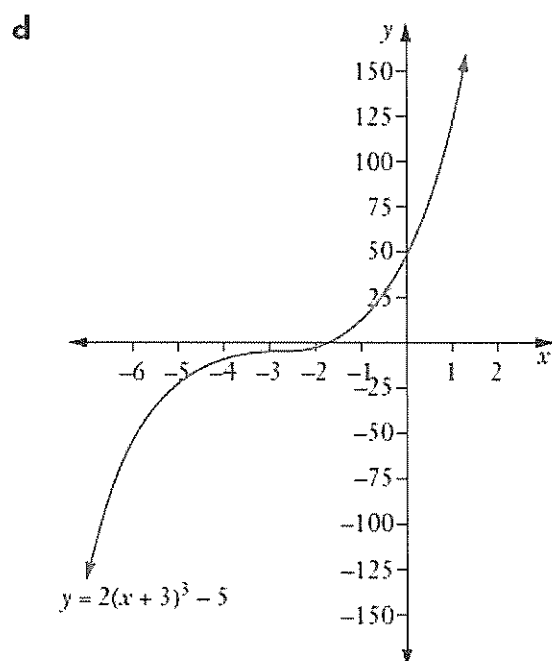
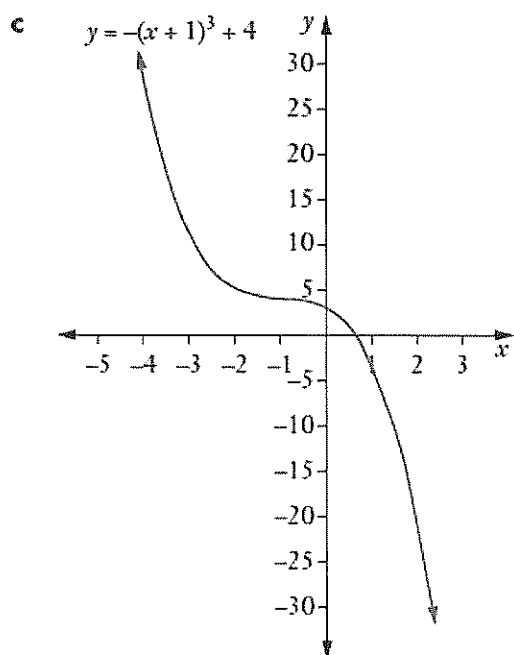
- 1** **a** $(5, -11)$ **b** $(-1, -2)$
c $(9, 3)$ **d** $(-2, -17)$
- 2** **a** $f(x) = -4x^5$ **b** $f(x) = -\frac{1}{243}x^5$
- 3** **a** $y = (x + 4)^3 - 3$ **b** $f(x) = |x - 1| + 9$
c $f(x) = 3x - 6$ **d** $y = -e^x + 2$
e $y = (2x)^3 - 5$ **f** $f(x) = \frac{6}{x}$
- g** $f(x) = 3\sqrt{-2x}$ **h** $y = \ln \frac{x}{3} + 2$
- i** $f(x) = 3 \log_2 4x$ **j** $y = \left(\frac{x}{2}\right)^2 - 3$
- 4** **a** Horizontal translation 1 unit to the right, vertical translation 7 units up
b Vertical dilation scale factor 4, vertical translation 1 unit down
c Vertical dilation scale factor 5, reflection in x -axis, vertical translation 3 units down
d Horizontal translation 7 units to the left, vertical dilation scale factor 2
e Rewrite as $y = 6[2(x - 2)]^3 + 5$. Horizontal dilation scale factor $\frac{1}{2}$, horizontal translation 2 units to the right, vertical dilation scale factor 6, vertical translation 5 units up
f Rewrite as $y = 2[3(x + 3)]^3 - 10$. Horizontal dilation scale factor $\frac{1}{3}$, horizontal translation 3 units to the left, vertical dilation scale factor 2, vertical translation 10 units down
- 5** **a** Horizontal translation 3 units to the left, vertical dilation scale factor 2, vertical translation 1 unit down
b Horizontal dilation scale factor $\frac{1}{3}$, reflection in x -axis, vertical translation 9 units up
c Horizontal dilation scale factor $\frac{1}{5}$, vertical dilation scale factor 2, vertical translation 3 units down
d Horizontal translation 7 units to the right, vertical dilation scale factor 4, vertical translation 1 unit up
e Reflection in y -axis, horizontal dilation scale factor $\frac{1}{2}$, horizontal translation 1 unit to the left, vertical translation 1 unit down
- f** Horizontal dilation scale factor $\frac{1}{2}$, reflection in x -axis, vertical translation 8 units up
- 6** **a** $(9, -31)$ **b** $(4, 5)$
c $(5, -25)$ **d** $(-8, -67)$
e Change to $y = -2f[2(x - 2)] - 3$: $(6, 21)$
- 7** **a** $(x + 3, y - 6)$ **b** $(-x, y + 6)$
c $(x - 5, 2y)$ **d** $(3x, y + 5)$
e $(5x - 30, -8y - 1)$
- 8** **a** $y = f(x + 1) - 2$ **b** $y = f(x - 5) + 3$
c $y = -f(x - 4)$ **d** $y = f(-x) + 2$
e $y = -f\left(\frac{x}{4}\right)$ **f** $y = 2f(x) - 2$
- 9** **a** $f(x) = -\frac{9}{x} + 3$ **b** $y = 5(x + 2)^2 - 6$
c $f(x) = 8 \ln \left[\frac{1}{2}(x - 5) \right] - 3$
d $y = 9\sqrt{-(x + 4)} + 4$ **e** $f(x) = -|6x| + 7$
f $y = [4(x + 4)]^3 = 64(x + 4)^3$
g $y = 6(2^{x-2} + 5)$
- 10** **a** Domain $(-\infty, \infty)$, range $[5, \infty)$
b Domain $(-\infty, \infty)$, range $[-2, \infty)$
c Domain $(-\infty, 2) \cup (2, \infty)$, range $(-\infty, 1) \cup (1, \infty)$
d Domain $(-\infty, \infty)$, range $(2, \infty)$
e Domain $(2, \infty)$, range $(-\infty, \infty)$
- 11** **a** $y = (x + 1)^2 - 8$
b Horizontal translation 1 unit to the left, vertical translation 8 units down
- 12** Horizontal translation 5 units to the right, vertical translation 28 units down
- 13** **a** $(2x + 3, 2y + 5)$ **b** $\left(\frac{x}{3} - 6, -y - 2\right)$
- 14** **a** Circle $(x - 3)^2 + (y - 4)^2 = 9$ or $x^2 - 6x + y^2 - 8y + 16 = 0$
b Translated 2 units to the right, 3 units down

Exercise 2.06

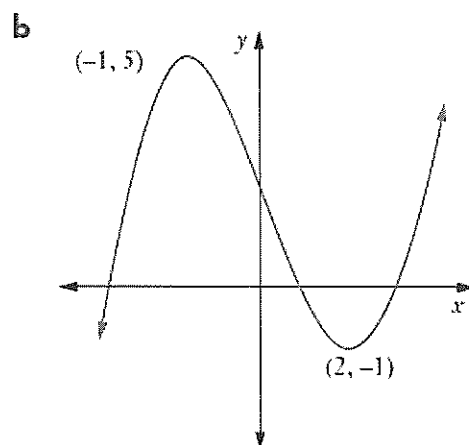




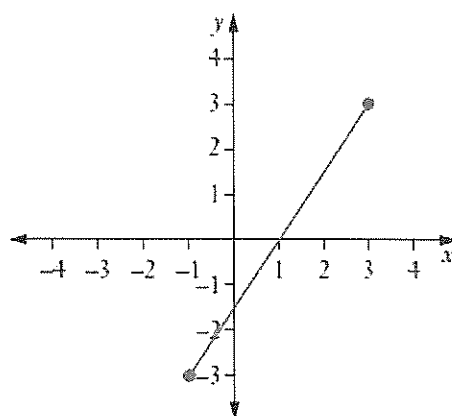




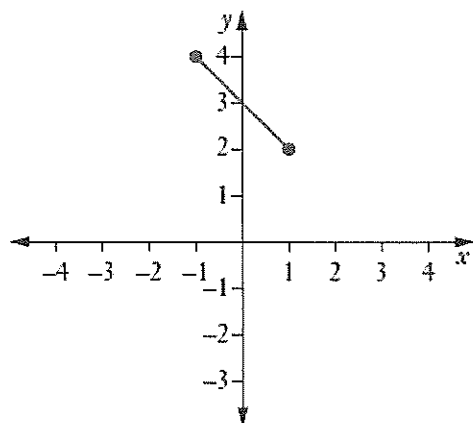
4 a (2, -1) and (-1, 5)



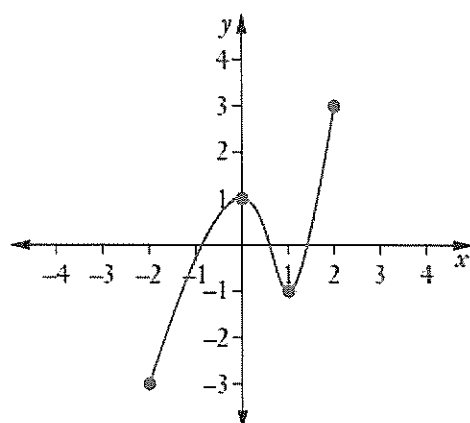
5 a i



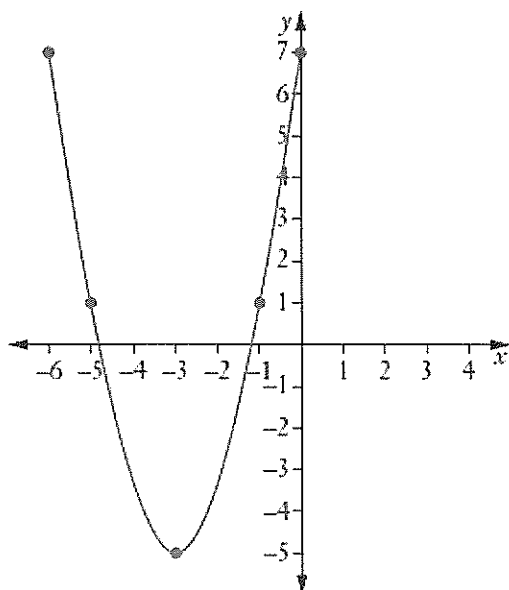
ii



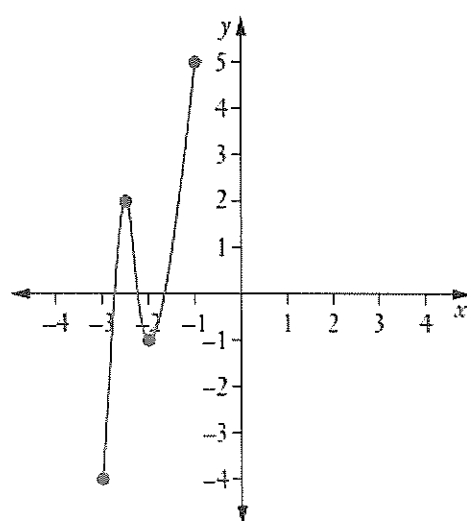
c i



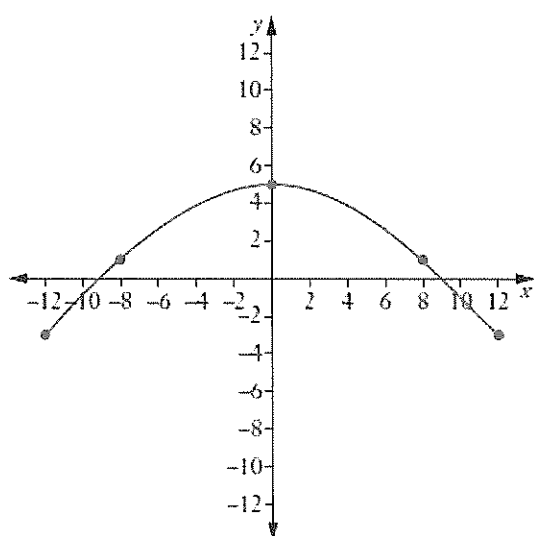
b i



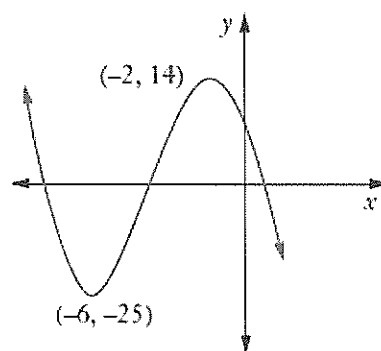
ii



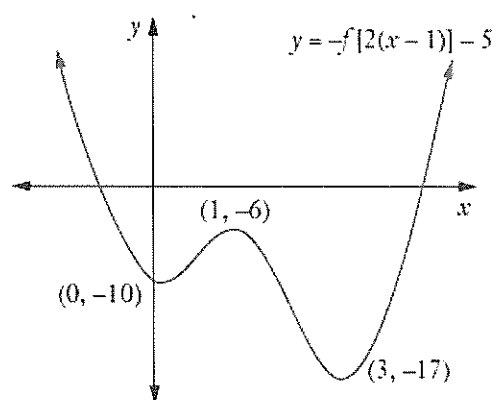
ii



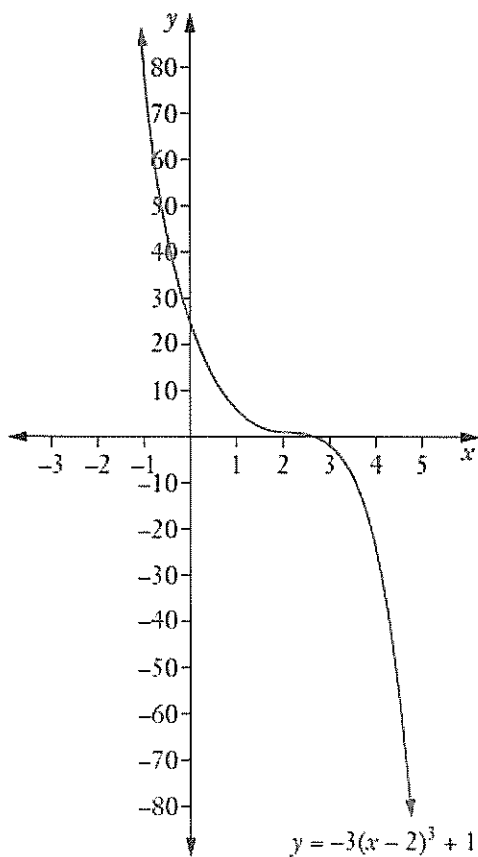
6



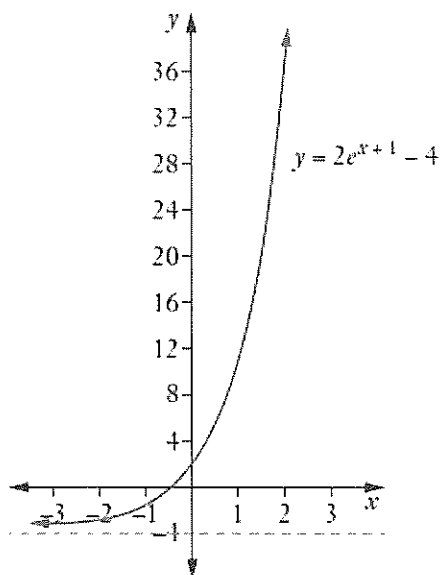
7



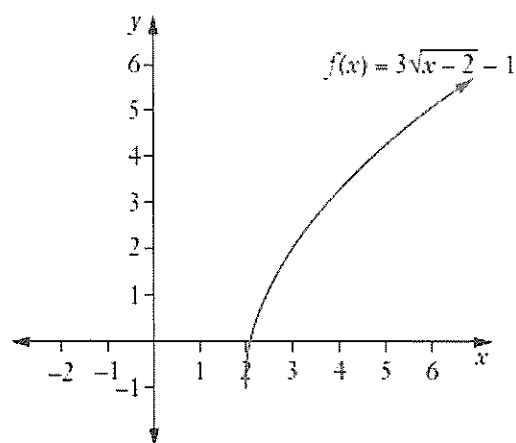
8 a



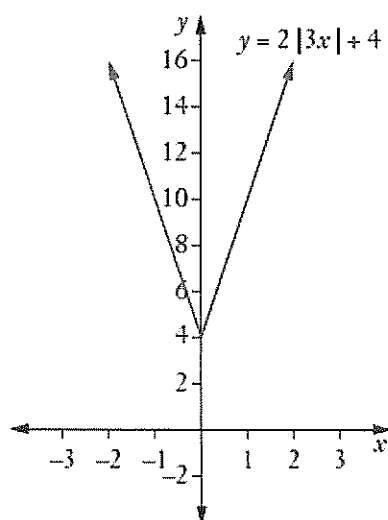
b



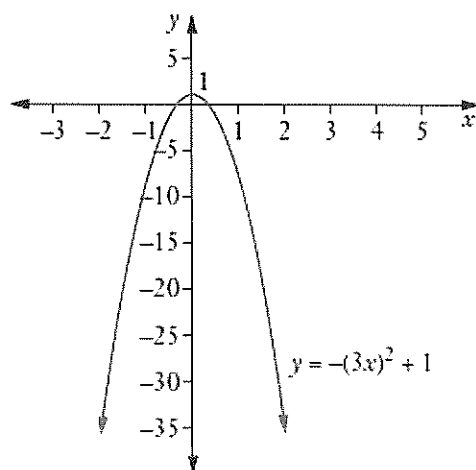
c



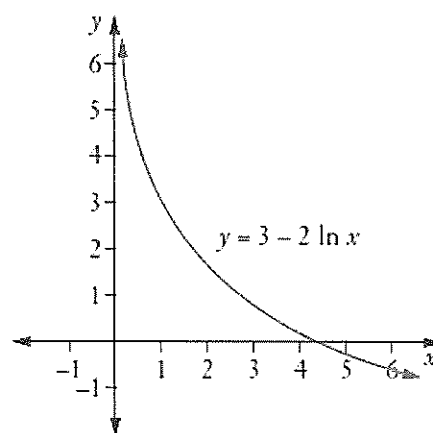
d

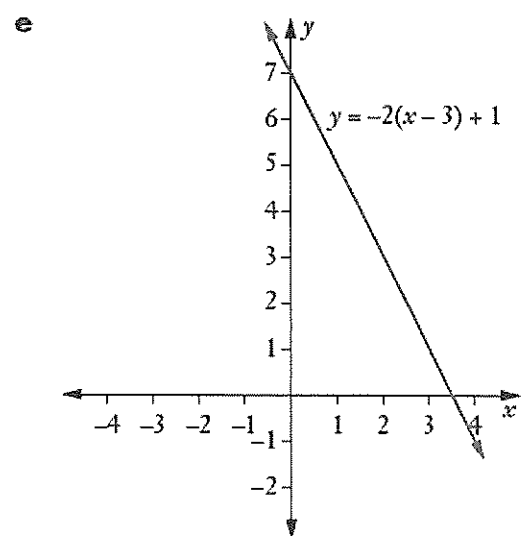
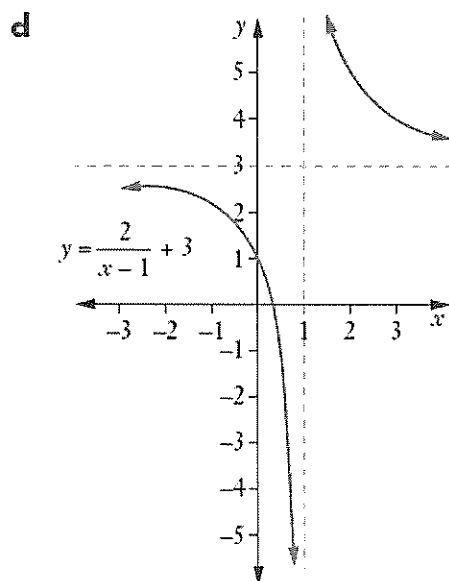
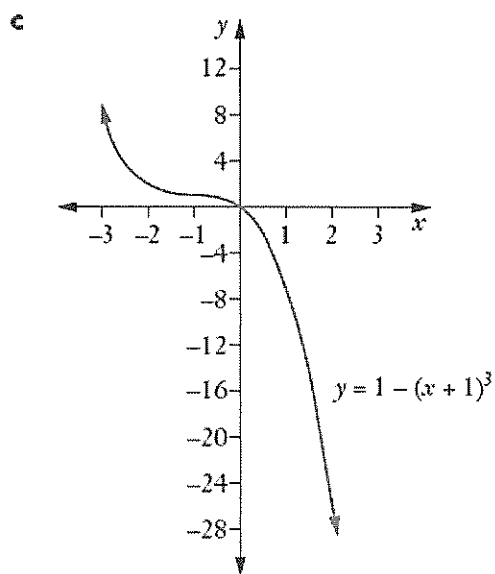
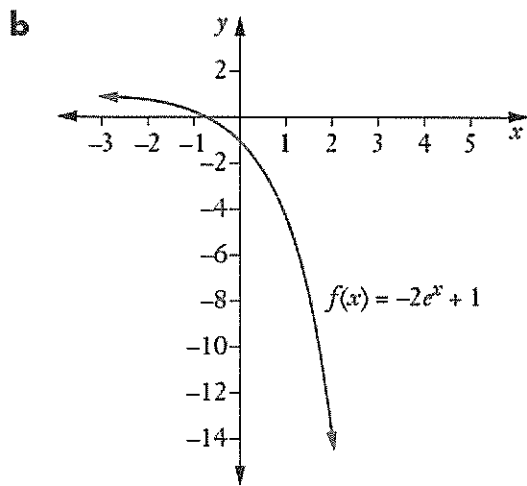


e

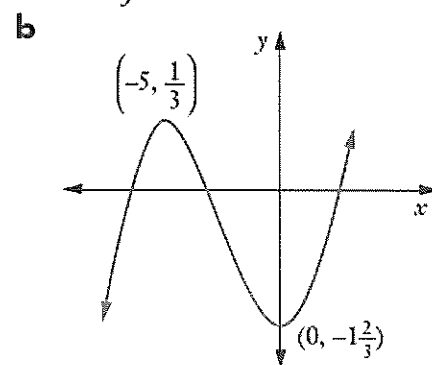


9 a

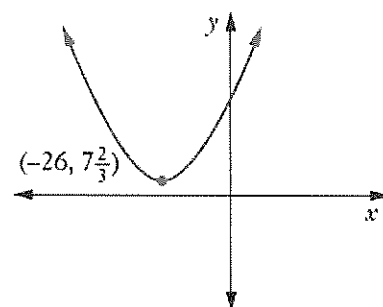




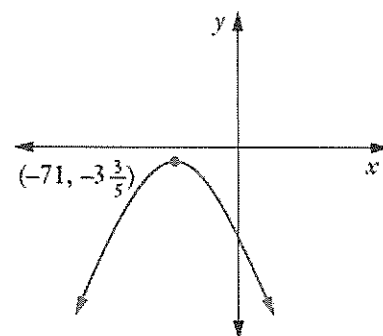
10 a $(-5, \frac{1}{3})$



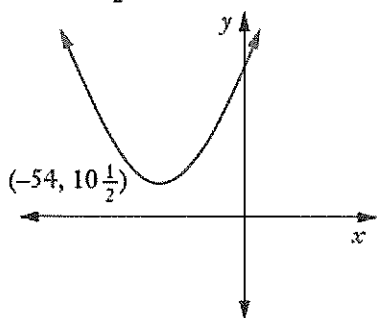
11 a $(-26, 7\frac{2}{3})$



b $(-71, -3.6)$



c $(-54, 10\frac{1}{2})$



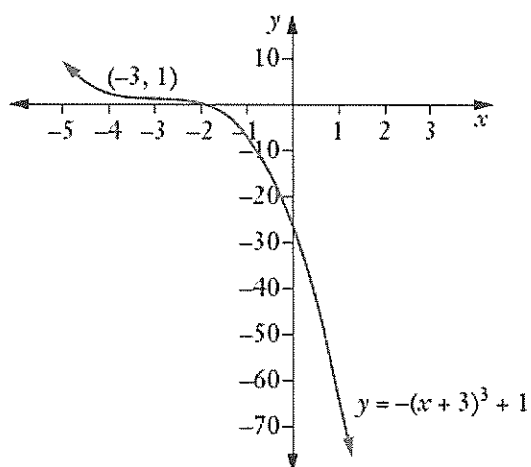
Exercise 2.07

- 1 a 2 b 0 c 1 d 3
 e 0 f 1 g 2 h 0
 i 1 j 0

- 2 a i $x = -2, 0$ ii $x = 0.6, -2.6$
 iii $x = -2.2, 0.2$ b $x = 0.2, -2.2$

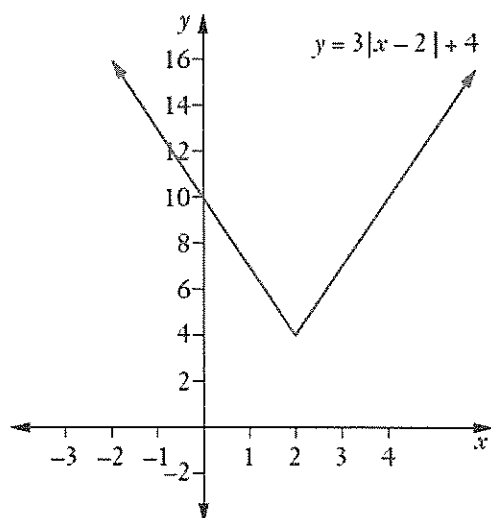
- 3 a $x = 1.4$ b $x = 1.9$ c $x = 0.2$
 d $x > 2.2$ e $x \leq 3.1$

4 a



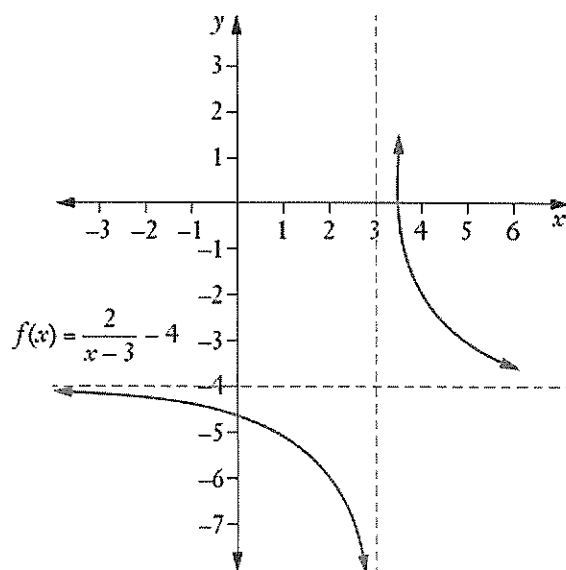
- b i $x = -2$ ii $x = -0.8$ iii $x = -0.3$
 c $x = -2$

5 a



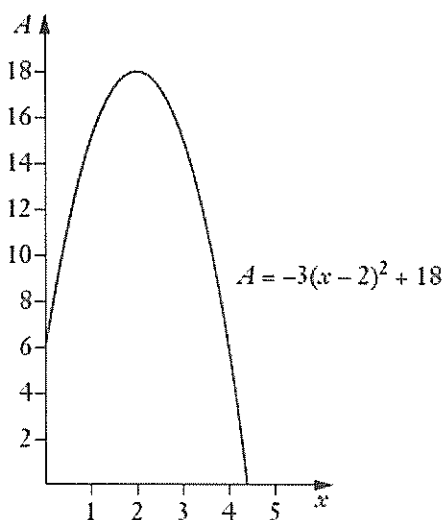
- b None c $x = 0, 4$

6 a



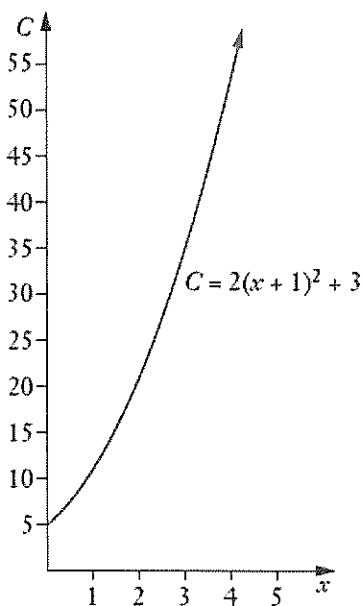
- b $x = 1$ c $x = 4$

7 a



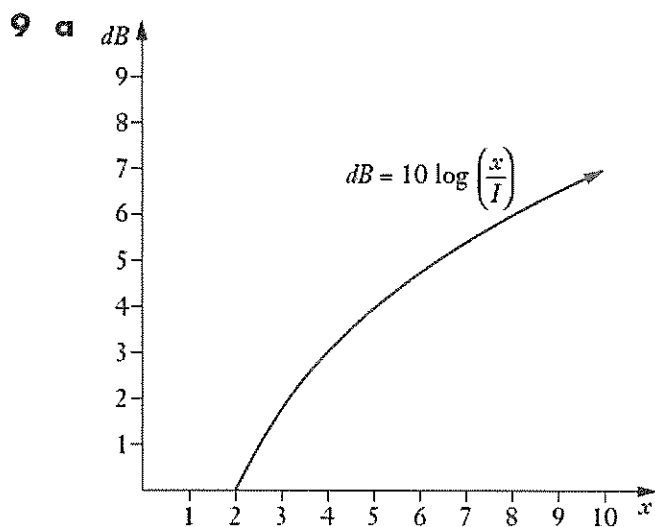
- b $x = 0.4, 3.6$

8 a



b \$5000

c The costs are \$20 000 when 2 products are made.



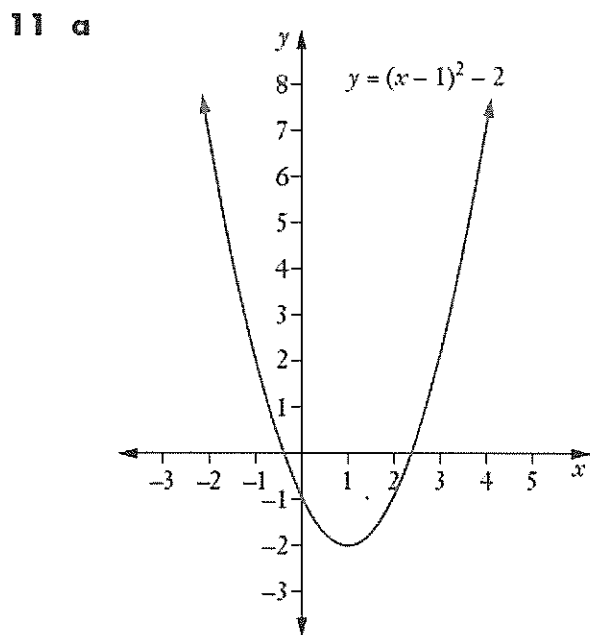
b i $x = 6$ **ii** $x = 3$

10 a i $t = 3.5$. After 3.5 minutes the temperature is 50°C .

ii $t = 8$. After 8 minutes the temperature is 30°C .

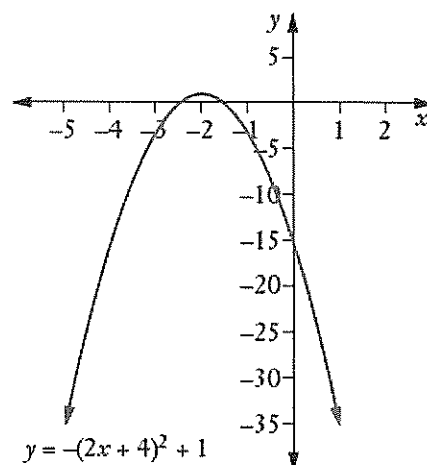
b i $t = 0.74$ **ii** $t = 11.85$

c 24°C (room temperature)



b i $x = 3, -1$ **ii** $x \leq -1, x \geq 3$
iii $-1 < x < 3$

12 a



b i $x = -3, -1$ **ii** $-3 < x < -1$
iii $x \leq -3, x \geq -1$

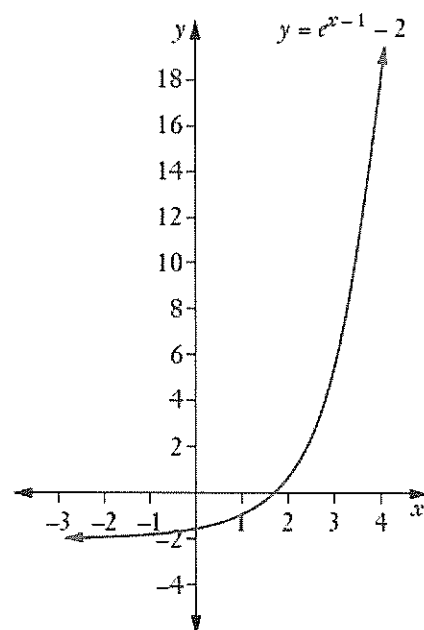
Test yourself 2

1 B

2 D

3 C

4 a



b $x = 3.1$

c $x = 4.09$

5 a (6, 107)

b (6, 40)

c (-24, 177)

d (17, -75)

e (16, -31)

6 a i $y = x^3 + 3$

ii $y = (x+7)^3$

b i $y = 3|x|$

ii $y = \left| \frac{x}{2} \right|$

c $f(x) = 5 \ln(-x)$

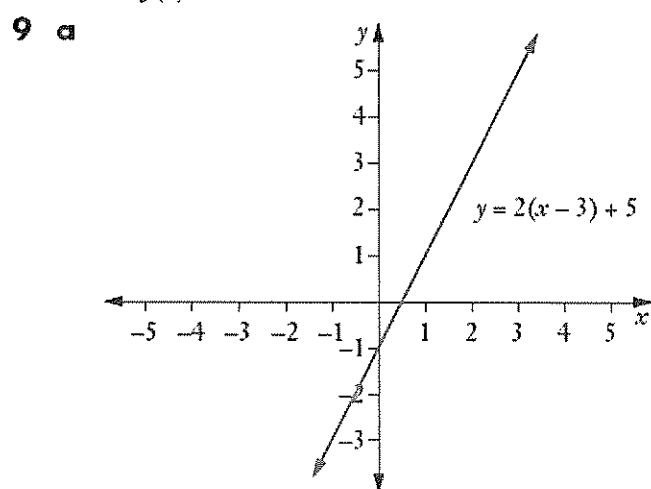
d $f(x) = -\frac{1}{x-4}$

e $f(x) = 9 \cdot 3^{3x-2} - 6$

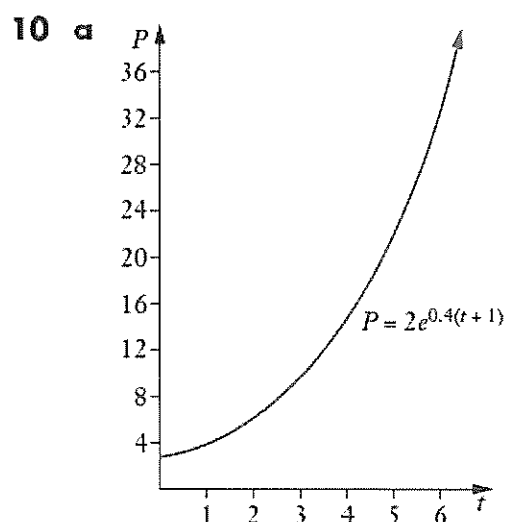
- 7 a** c is a vertical translation c units up if $c > 0$ or down if $c < 0$.
 b is a horizontal translation b units to the right if $b < 0$ and to the left if $b > 0$.
 k is a vertical dilation with scale factor k , stretched if $k > 1$ and compressed if $0 < k < 1$.
 a is a horizontal dilation with scale factor $\frac{1}{a}$, compressed if $a > 1$ and stretched if $0 < a < 1$.

- b i** Reflection in the x -axis
ii Reflection in the y -axis

8 $f(x) = -3x^2 + 1$
 $f(-x) = -3(-x)^2 + 1$
 $= -3x^2 + 1$
 $= f(x)$ so even



- b i** $x \leq 4$ **ii** $x > 5$



- b** $t = 1.3$. It takes 1.3 years for the population to reach 50 000.

11 $f(x) = (x + 4)^4$

12 $(3, -11)$

13 a $x = 2.88, 1.12$

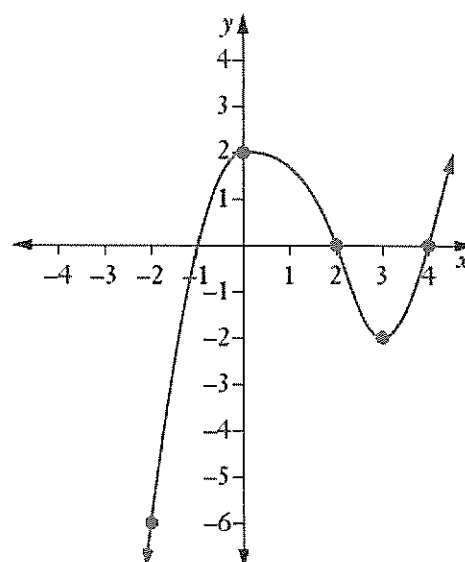
b $x < 1.12, x > 2.88$

c $1.12 \leq x \leq 2.88$

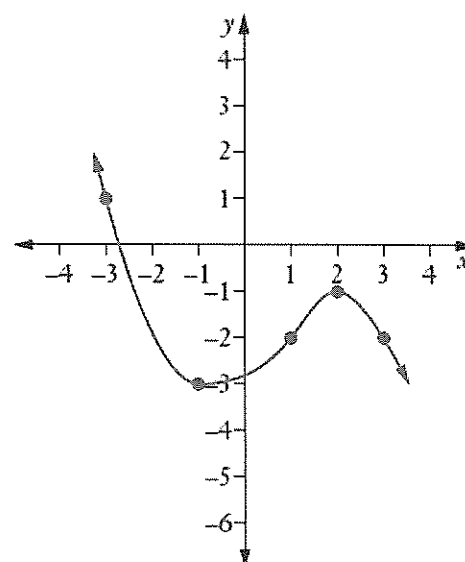
14 a $(x + 3, -7y - 4)$

b $x = -6, y = -1$

15 a



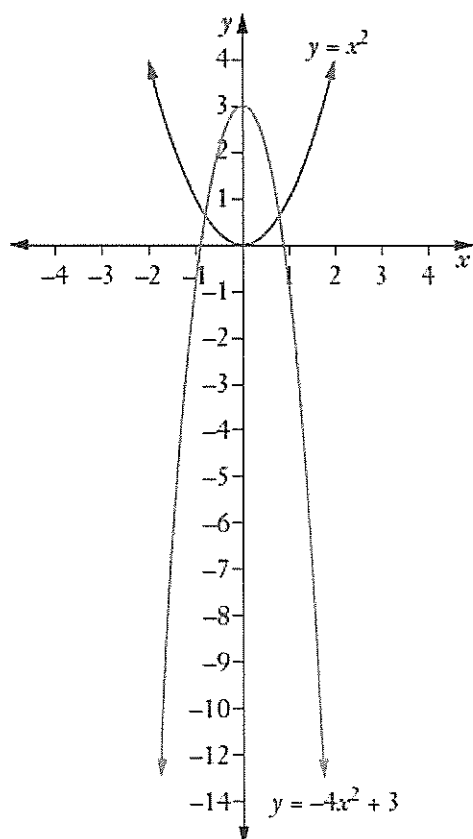
b



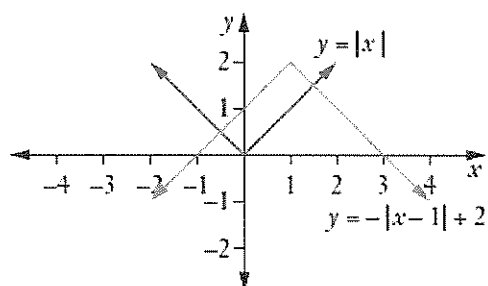
16 a $-3 \leq x \leq 1$

b $x < -3, x > 1$

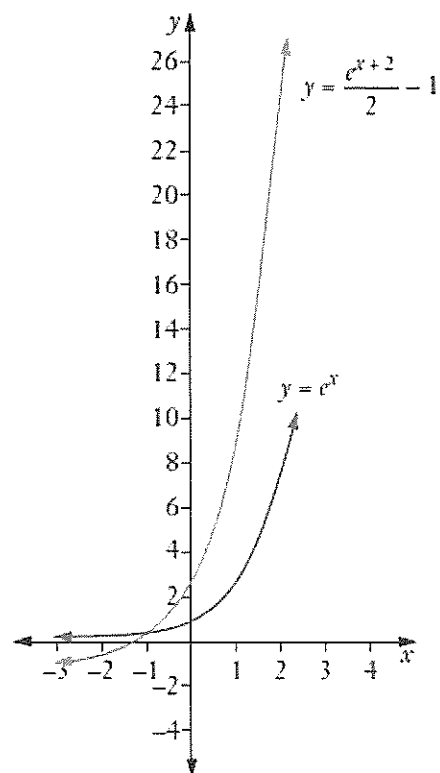
17 a



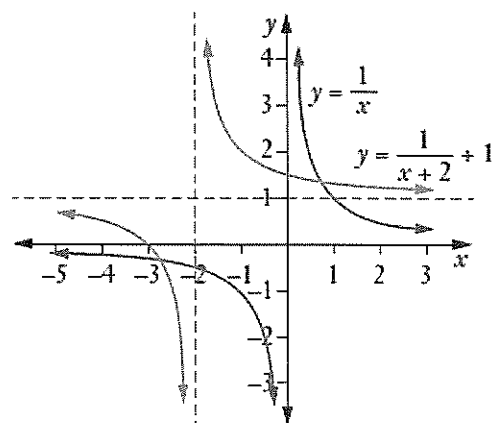
b



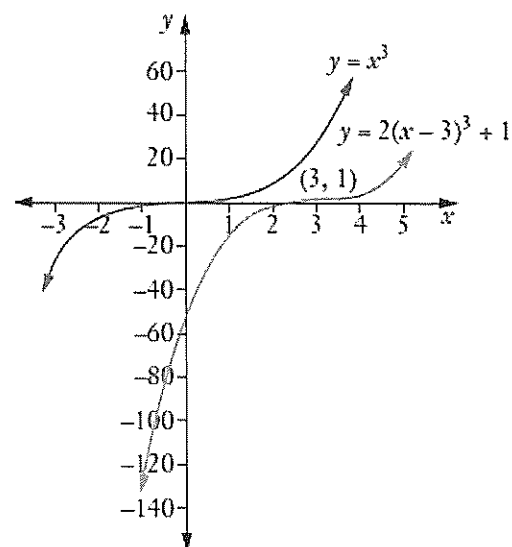
c



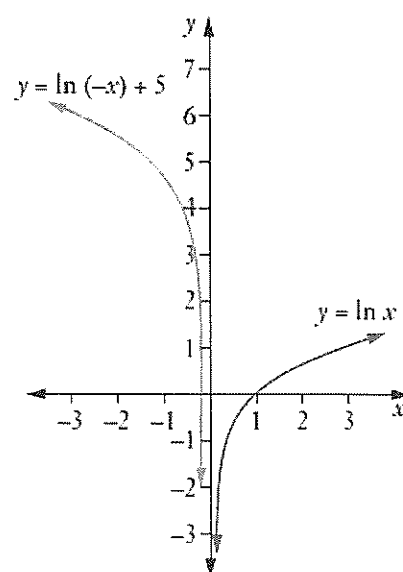
d



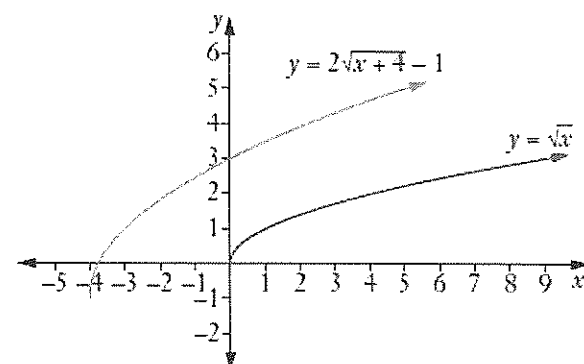
e



f

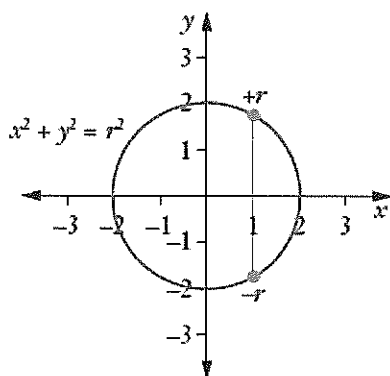


g



- 18 a 2 b 1 c 0 d 1
 e 1 f 0 g 1 h 0
 i 4 j 3

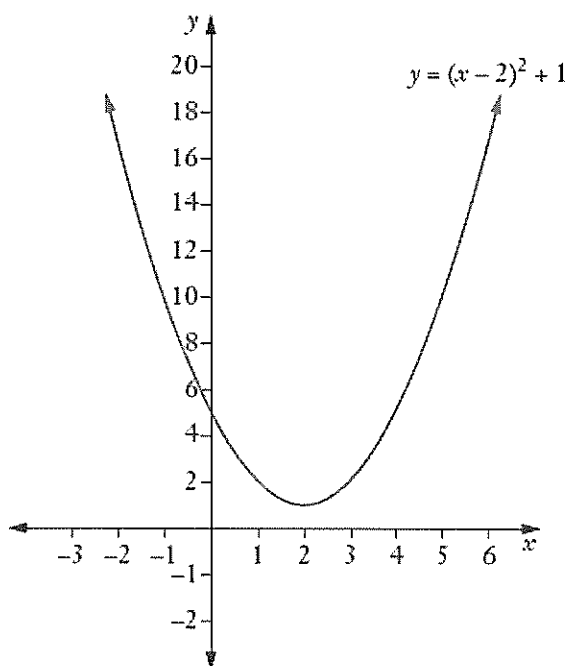
- 19 a The circle is not a function since a vertical line cuts the graph in more than one point.



- b $y = \sqrt{r^2 - x^2}$ and $y = -\sqrt{r^2 - x^2}$
 c An ellipse (oval)

- 20 (32, -1)

- 21 a



- b i $x = -1, 5$ ii $x < -1, x > 5$
 iii $-1 \leq x \leq 5$

- 22 a $(x - 1, 3y - 5)$ b $\left(\frac{1}{2}[x + 6], -2y + 4\right)$

- c $(-x, 5y - 3)$ d $\left(-\frac{1}{3}[x + 3], -3y - 1\right)$

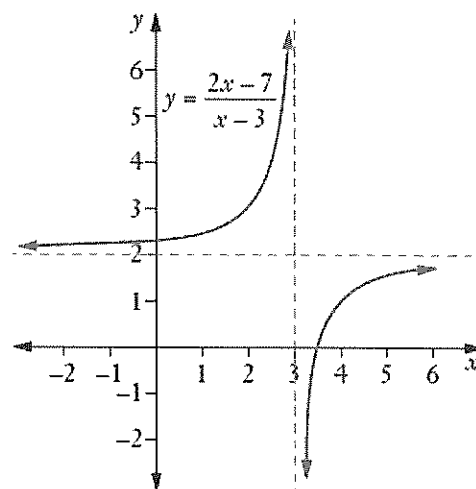
- 23 a Stretched b Compressed
 c Stretched d Compressed
 e Stretched

- 24 a Domain $(-\infty, \infty)$; range $[-10, \infty)$
 b Domain $(-\infty, \infty)$; range $(-\infty, 2]$
 c Domain $(-\infty, 3) \cup (3, \infty)$;
 range $(-\infty, -5) \cup (-5, \infty)$

Challenge exercise 2

- 1 a $h = -2t^2 + 4t + 1$ b 2.2 seconds
 c $h = -2(t - 1)^2 + 3$. Horizontal translation 1 unit to the right, reflection in the x -axis, vertical dilation scale factor 2, vertical translation 3 units up.
 2 a i (1, -8) ii (2, 0) iii (3, -2)
 b $x - 2y - 17 = 0$
 c Horizontal dilation with scale factor 2 and horizontal translation 17 units to the right,
 OR: vertical dilation with scale factor $\frac{1}{2}$
 and vertical translation $\frac{17}{2}$ units down

- 3 b



- c i $x < 3, x \geq 3.5$ ii $x > 3$

- 4 a A horizontal dilation with scale factor $\frac{1}{a} = 2$:

$$a = \frac{1}{2}$$

$$y = \frac{1}{(ax)} = \frac{1}{\frac{1}{2}x} = \frac{2}{x}$$

A vertical dilation with scale factor $k = 2$:

$$y = \left(\frac{1}{x}\right) = 2\left(\frac{1}{x}\right) = \frac{2}{x}$$

So these transformations have the same effect on $y = \frac{1}{x}$.

- b No. Horizontal dilation gives $y = \frac{4}{x}$, vertical dilation gives $y = \frac{2}{x}$.

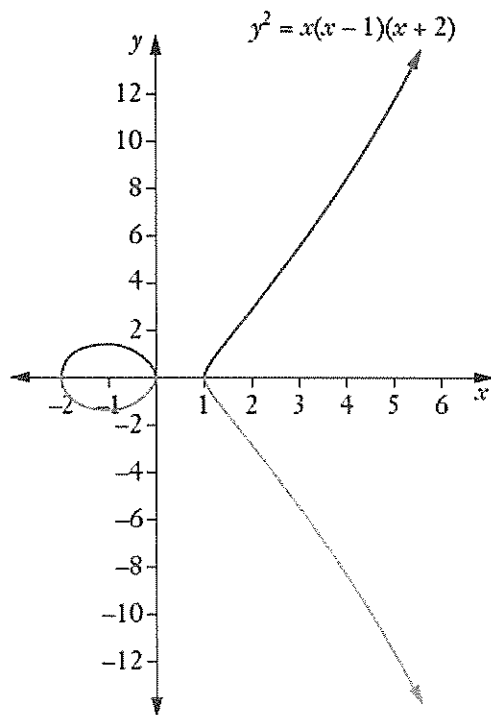
5 a $x = -\frac{b}{2a}$ b Horizontal translation

c i $x = -1$ ii $x = 3$
 iii $x = -b$ iv $x = -\frac{b}{a}$

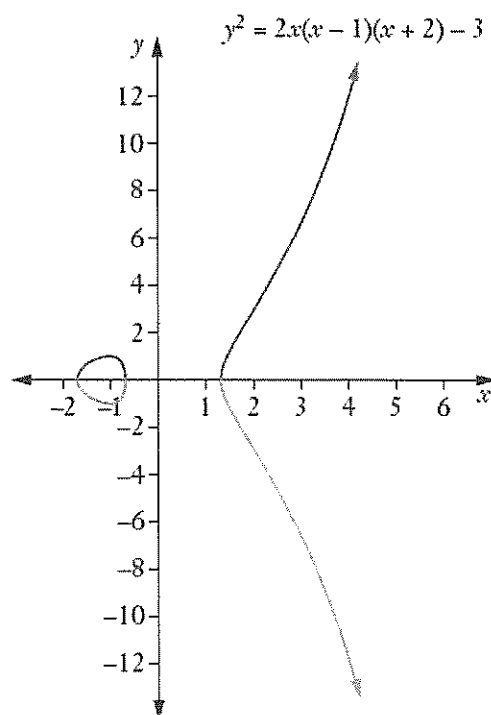
6 a $y = -3 \sin \frac{x}{2} - 1$

b Amplitude 3, period 4π , centre -1

7 a



b



8 $x^2 - 6x + y^2 + 8 = 0$

9 Reflection in y -axis, horizontal dilation scale factor $\frac{1}{3}$, horizontal translation 2 units to the left, vertical dilation scale factor 3, vertical translation 5 units down

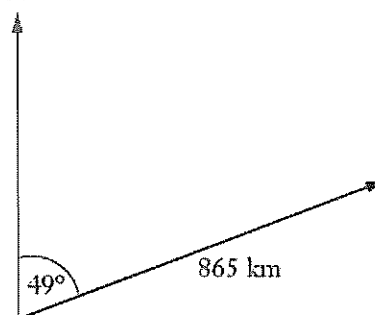
10 $y = -x^3 + 3x$

Chapter 3

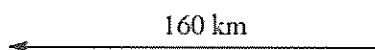
Exercise 3.01

- 1 a Scalar b Vector c Vector
 d Vector e Scalar

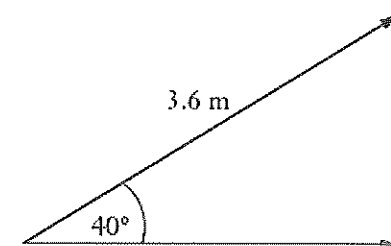
2 a N



b



c



d

